# The Optimal Portfolio of PAYG Benefits and Funded Pensions in Germany*

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**Abstract**

The combination of a pay-as-you-go (PAYG) and a self-funded pension system is studied from a portfolio perspective considering the tradeoff between speculation and hedging. We analyze the German PAYG social security system and subsidized private savings (Riester pension). The return and the risk associated with a combination of both systems is simulated in a stochastic economy. Our results suggest that (1) a risk-minimizing structure of total retirement income still allows a fraction of 5 percent to be financed via private savings due to benefits of hedging, (2) the optimal portfolio shares at a fixed total pension level implies high fractions of private savings even if agents are highly risk-averse, and (3) determining the optimal size of the total pension level under the current social security system implies only low additional private saving rates.

**Keywords**: Social security, PAYG system, funded pensions, optimal portfolio

**JEL codes**: H55, C53, G11

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# 1 Introduction

In the past decades, reforms of the PAYG social security system in Germany aimed at strengthening the fully funded pillar of retirement savings and reforming the social security system to keep it financially sustainable. As two prominent examples, a subsidy to private savings, the so called Riester pension plan, was introduced in 2001 and in 2008 the government passed a law that increased retirement age to 67. These reforms were driven by the observation that the ageing of the German society and continuously low fertility rates put an increased pressure on financing the current PAYG social security system. More recently, however, the German government has implemented a series of reforms that again expanded the PAYG social security system.[1] According to a new concept of the government (BMAS, Das Gesamtkonzept zur Alterssicherung, 2016), it is planned that the total benefits level shall not fall below 46 percent until 2045 while the rise of the contribution rate is capped at 25 percent. Due to the ongoing demographic change, an implementation of both limits leads to an increased budget deficit of the social security system.

Despite the argument that additional private retirement savings seem to be inevitable to keep a PAYG system financially sustainable in an ageing society, it is often argued that this would even be beneficial for the population due to higher returns at the stock market. Demographic change reduced the implicit return of the German PAYG system to very low values of around one percent for the last years (Schnabel, 1998). On the contrary, investing private retirement savings in stocks and government bonds yield high expected returns: The 40-year-average rate of return of the German stock market index (DAX) is around 5.5 percent. However, private savings invested at the stock markets are much more volatile implying that the higher returns come at a cost of higher risk. This concern might be even more important once the recent financial crisis is taken into account.

This paper studies the PAYG system and a privately funded system from a portfolio point of view. We interpret both systems as assets that can be combined as a portfolio. The PAYG system can be interpreted as a riskless-low-return asset while the privately funded pillar is associated with high returns and higher risk.

[1] Important reforms that have been adopted are the extension of early retirement possibilities and a special support for mothers who stayed at home for some time during working life.

The risks associated with the PAYG system are dominantly driven by demographic developments of a country. In upcoming years there will be lesser contributors into the system and more retirees. In addition, in Germany public benefits of the PAYG system are linked to labor supply and overall wages and in this respect depend on the business cycle. Hence, the PAYG system is also affected by shocks in the economy. A funded system is predominantly prone to financial risks and hinges only to a lesser degree on demographic change.[2] The risk is mainly driven by fluctuating returns at the capital markets. A combination of both systems has the potential to hedge the demographic risk of the PAYG system and the financial risk of a funded system. Merging the two risks might reduce overall variance as the two 'assets' are exposed to different kinds of risks.

We model the special aspects of the German social security system with its main pillar, the PAYG system, as well as the privately funded pillar, the Riester pension plan. The framework is stochastic, where GDP exhibits business cycle fluctuations and the rate of return at the capital market fluctuates. In addition, we incorporate "rare disasters" to study the impact of stock market crashes like the current crises on the privately funded part. The stochastic processes are matched to moments from past data. In this framework we evaluate the optimal share of these two assets from a portfolio point of view. To this end, we determine the portfolio which leads the maximal return under the minimal variance which is equivalent to an optimum for an infinitely risk-averse investor. We also compute the optimal portfolio within a life-time utility framework with various degrees of risk aversion.

As our first main finding, we report that even if it is the aim to minimize overall risk, a positive share of funded benefits of 5 percent is needed for a portfolio with minimum variance. Hence, a positive fraction of private savings for retirement simultaneously increases the implicit rate of return of the portfolio *and* decreases risk compared to a pure PAYG system. However, maintaining the pension benefit level at 70 percent requires an average contribution rate of almost 37 percent due to demographic change.

Employing a concept that maximizes lifetime utility of agents that can still adjust via private saving we find that the optimal share of the PAYG benefits are rather low for reasonable risk aversion parameters. But even with high risk aversion, the share of PAYG system in an optimal portfolio is less than two third. However, if we instead determine the optimal additional savings

[2] Of course, demographic change alters the factor prices in a general equilibrium framework, see Krueger and Ludwig (2007).

taking the current social security system in Germany as given, we find that risk averse agents are willing to forego a significant fraction of retirement income in order to avoid the risks associated with higher private savings. The optimal pension benefit level is 78 percent assuming a risk-aversion parameter of three.

The paper proceeds as follows. The next section nests our work within the existing literature. Section 3 outlines the simulation and Section 4 describes the calibration procedure. In Section 5 we present our result and perform a sensitivity analysis in Section 6. Finally, we conclude in Section 7.

# 2 Literature Review

The classical argument in favor of a PAYG benefit system is based on the possibility of dynamic inefficiency where the implicit return of the PAYG system is lower than the interest rate, cf. Diamond (1965). In this case, the economy suffers from capital overaccumulation where individuals are saving too much. A PAYG benefit system is a device to redistribute consumption across generations and restoring efficiency. However, a dynamically efficient economy is seen as the more realistic case by most scholars.[3] Moreover, it is argued that a PAYG system distorts labor supply decisions and crowds out private savings, see Lindbeck and Persson (2003) which would call for a transition to a fully funded system to be beneficial.

The above argument in favor of a funded system do not take into account, however, that the potential efficiency gains from a shift to a funded system might be lost once transitional dynamics are taken into account. For a transition to be welfare improving, the generation that contributed into the system and is about to retire, needs to be compensated. In addition, the two systems imply very different inter- and intra-generational redistributional consequences: a fully funded system might imply lower redistribution compared to a publicly provided PAYG system.[4]

An important aspect that is missing in the arguments above and that we highlight in our analysis is the fact that a PAYG system and fully funded pensions comprise a very different risk structure. If taking into account that the returns of the two systems are stochastic and the systems

[3] See von Weizsäcker (2016) for an exception to this view.
[4] See Lindbeck and Persson (2003), Breyer (2003), and Börsch-Supan (2005) for an overview of the arguments on a mix of a PAYG and a funded system.

differ in their degrees of risk, a portfolio consisting of the PAYG pillar together with a fully funded part with higher expected returns in the stock market might be preferable.

This paper adds to the still rather small literature on the study on the optimal mix of PAYG and funded social security system under risk. Persson (2000) explains the potential to hedge with the two systems verbally and presents simple numerical illustration based on Swedish data. Dutta, Kapur und Orszag (2000) and Matsen und Thøgersen (2004) show formally that the diversification of the risks of the two systems leads to utility improvements if the correlation of the two assets is low.[5] Most closely related to our paper are Nataraj and Shoven (2003) for the US and Broer, Knaap and Westerhout (2010) for the Netherlands. The former simulate returns of different portfolios of the two social security systems for the US and show how the optimal portfolio depends on the risk aversion of the household. Concentrating on the US they find that for reasonable degrees of risk-aversion a fully funded system maximizes utility. On the other hand, Broer, Knaap and Westerhout (2010) do not calculate the optimal portfolio but rather concentrate of the importance of different kinds of risk.

We add to this literature by analyzing the German social security system in detail with its new self-funded pillar, the Riester pension plan. We study the optimal share of PAYG benefits and funded pensions by means of different assumption about risk-aversion. In addition, we take the current benefit formula in Germany as given and determine an optimal saving rate – and hence a total benefit level – that maximizes lifetime utility. Finally, we incorporate "rare disasters" into the simulation to be able to study the effects of a shock like the financial crisis of 2007.

Our analysis focus on an optimal share of unfunded benefits and funded pensions from a portfolio point of view. We abstract from various other sources of risk that are important and to which social security provides (partly) insurance: labor productivity risks, marital risks, and especially survival risk. All these risks might be better insured in a publicly provided social security system compared to the market.[6]

[5] Our analysis is also related to general equilibrium models where the positive risk-sharing effects of social security is valued against the negative crowding-out effect, e.g. Storesletten, Telmer and Yaron (1999) or Krueger and Kubler (2006). These studies generally model the pension systems in a stylized way and do not derive a specific share of each system depending on different concepts of risk.

[6] See Groneck and Wallenius (2016).

# 3 Optimal Portfolio of Retirement Income under Uncertainty

## 3.1 Simulation Procedure

The optimal portfolio of PAYG benefits $b_t$ and pensions from a privately funded system $f_t$ is simulated in a framework with stochastic GDP growth and factor prices as well as the projected demographic transition. We assume perfectly rational agents that chose their optimum according to the conditions outlined below.[7]

We conduct two different simulations:

1. We determine the optimal portfolio as an average of all years, fixing the ratio of total retirement income to average gross wages, $b_t^{tot}$.
2. Given the current system in Germany, we simulate the evolution of social security benefits $b_t$ and of the contribution rate $\tau_t$, and determine the optimal ratio of total retirement income by finding the optimal saving rate.

### 3.1.1 Optimal Portfolio

Determining the optimal portfolio requires to fix the total gross pension level. The gross pension level is defined as the total retirement income, i.e., the sum of PAYG benefits $b_t$ and privately funded annuities $f_t$ relative to actual average gross wages $w_t$

$$b_t^{tot} = \frac{b_t + f_t}{w_t}. \tag{1}$$

We assume the ratio of total retirement income relative to gross wages to be fixed at $b_t^{tot} = 0.7$. The optimal portfolio is determined by the optimal share of total benefits α that is financed via PAYG benefits, such that $b_t = \alpha b_t^{tot} w_t$ and$f_t = (1-\alpha) b_t^{tot} w_t$, where the time series for wages is simulated. Note, that for this exercise, we do not apply the current German benefit formula. Instead, $b_t$ is endogenously determined by the optimal portfolio share which we aim to determine. The corresponding adjustments of the PAYG contribution rates for each fraction α are calculated using the social security budget constraint, similar to Wilke (2004). Private

[7] See Laibson et al. (1998) for the importance of undersaving for retirement due to hyperbolic discounting.

savings are assumed to be invested in a *Riester pension plan* where the institutional details of these products are taken into account when simulating the rates of return.

Let $i^P$ be the internal rate of return of the PAYG system and $i^F$ the corresponding return of the Riester-plan, where both returns $i^P$ and $i^F$ are random variables, cf. subsection 3.4 for how we calculate the internal rate of returns. The expected total return of a portfolio is then given by

$$E[i_\alpha] = E[\alpha \cdot i^P + (1-\alpha) \cdot i^F], \tag{2}$$

where $0 \leq \alpha \leq 1$ determines the fraction of social security benefits $b_t$ and $(1-\alpha)$ the corresponding fraction of private savings in Riester products $f_t$. A risk-neutral agent would simply choose the portfolio that delivers the highest expected return irrespective of its variance. The Variance of the portfolio is given by

$$\sigma_\alpha^2 = \alpha^2\sigma_P^2 + (1-\alpha)^2\sigma_F^2 + 2\alpha(1-\alpha)\sigma_P \cdot \sigma_F \cdot \rho_{P,F}, \tag{3}$$

where $-1 \leq \rho_{P,F} \leq 1$ is the correlation coefficient between the internal rates of returns of the PAYG system and of private savings. The combinations of $i_\alpha$ and $\sigma_\alpha^2$ for each value of $\alpha$ determine the efficient frontier of portfolios. The portfolio theory proposed by Markowitz (1952) highlights the importance of $\rho_{P,F}$ for hedging possibilities: if $\rho_{P,F} = 1$ the rates of returns are perfectly correlated and hence there are no hedging possibilities. In contrast, if $\rho_{P,F} = -1$ the risk can be perfectly diversified by choosing $\alpha = \sigma_P/(\sigma_P + \sigma_F)$. For intermediate values, partial hedging is possible that give rise to an efficient frontier of potential portfolios. One can choose one optimum our of the set of efficient portfolios according to the Markov mean-variance formulation.

> **Definition: Maximal-return-under-minimal-variance.** (Markowitz, 1952) *A portfolio A is dominant over portfolio B if the return in A is at least as high as in B and the variance is lower, i.e.,*
>
> $$i_{\alpha_1} \geq i_{\alpha_2} \wedge \sigma_{\alpha_1}^2 < \sigma_{\alpha_2}^2 \quad or \quad i_{\alpha_1} > i_{\alpha_2} \wedge \sigma_{\alpha_1}^2 \leq \sigma_{\alpha_2}^2. \tag{4}$$

The optimal portfolio with respect to the mean-variance formulation corresponds to an optimum of an infinitely risk-averse agent who wishes an efficient portfolio with minimum risk. In order to account for in-between values of risk-aversion, we also consider a lifetime utility framework. In addition, by considering income streams over the entire life cycle the lifetime utility approach accounts for the tradeoff between the sacrifice of income during working life for higher income during retirement. To this end, we maximize a lifetime utility function with respect to the optimal share $\alpha$

$$\max_{\alpha} U = \sum_{t=t_0}^{T} \beta^{t-t_0} \cdot u(y_t^{\alpha}) \tag{5}$$

Per-period utility is given by a CRRA function

$$u(y_t^{\alpha}) = \frac{(y_t^{\alpha})^{1-\phi}-1}{1-\phi} \tag{6}$$

where $\phi$ is the (constant) measure of relative risk aversion. The agent enjoys utility from income $y_t^{\alpha}$ depending on the share of benefits $\alpha$:

$$y_t^{\alpha} = \begin{cases} w_t(1-\tau_t^{\alpha}) - s_t^{\alpha} & \text{for ages } t_0 \text{ to } t_{\mathrm{r}}-1 \\ b_t^{\alpha} + f_t^{\alpha} & \text{for ages } t_{\mathrm{r}} \text{ to } T. \end{cases} \tag{7}$$

where $t_{\mathrm{r}}$ is the first year of retirement and $T$ is the terminal age. During working age ($t_0$ to $t_{\mathrm{r}}-1$ the agent contributes $\tau_t^{\alpha} w_t$ to the social security system and invests $s_t^{\alpha}$ privately via Riester savings. During retirement $t_{\mathrm{r}}$ to $T$ the agent receives income from the PAYG system $b_t^{\alpha}$ and from private savings $f_t^{\alpha}$ where all variables depend on the share $\alpha$.

### 3.1.2 Optimal Total Pension Level

In a second exercise, we aim to study an optimal pension level, i.e. an optimal total retirement income ratio relative to gross wages. To this end, we take the current system in Germany as given and compute the evolution of benefits and contributions by means of the benefit formula and the assumption of balancing the social security budget constraint, cf. equation (14). Taken social security benefits as given by the current system, we can compute the amount of private savings in order to determine an optimal total pension level.

We adopt the same concept as outlined above with the difference that only the share of private savings $\bar{\alpha}$ can be chosen so that the expected return is given by: $i^P + \bar{\alpha} \cdot i^F$. Note, that this exercise determines the optimal total benefit level $b_t^{tot}$. However, $b_t^{tot}$ is bounded from below by social security benefits. Increasing private savings increases both the return and the variance of the portfolio. According to the optimal portfolio perspective, (i) returns are maximized with a maximal saving rate where the total ratio of retirement income $b_t^{tot} = 1$ and (ii) the maximal-return under minimum variance is found by choosing zero savings with $b_t^{tot} = b_t/w_t$. An interior solution for private savings exist when employing the lifetime utility concept with positive values of risk aversion by maximizing equation (5) with respect to $\bar{\alpha}$.

To determine the optimal pension level, we compute the time series for $b_t$ given by the benefit formula. We then determine private savings $f_t = \bar{\alpha} \cdot w_t$ that maximizes (5). Note, that $\bar{\alpha} \leq 1 - \frac{b_t}{w_t}$ because the total retirement income $b_t^{tot}$ cannot exceed one.

Our simulation for both exercises proceeds as follows: We compute time-series of GDP growth and interest rates from 2009-2068 where the former affects employment and wages in the economy. We then focus on one cohort born in 1979 and calculate the optimal portfolio and the total retirement income ratio taking into account the demographic transition. We run 5.000 Monte-Carlo simulations and then take a yearly average of the calculated optima to derive our main results.

The stochastic time series of GDP growth and the rate of return at the stock market are simulated as follows. For the rate of return we use the Vasicek (1977) model of the term structure extended with a Poisson-process to account for “rare disasters”, i.e. large shocks that hit the economy. For time series of GDP growth we employ the Kalman filter and use the estimated parameter to forecast GDP. The projected demographic process for Germany is taken from the Munich Center for the Economics of Aging (MEA). The demographic process and the simulated time series of GDP both determine total wages and employment. For the latter we estimate the elasticity of unemployment rates with respect to GDP growth.

## 3.2 Stochastic Economy

### 3.2.1 Rate of Return

We model the stochastic process for the rate of return $r_t$ using the Vasicek (1977) model which includes a drift term insuring that any deviation will eventually converge back to the long-run average $\mu_r$, where a parameter $\theta$ determines convergence speed. The process is defined as

$$r_t = r_{t-1} + \theta(\mu_r - r_{t-1}) + \sigma_r Z_t + J_r N_{r,t} \tag{8}$$

with the shock $Z_t \sim N(0, 1)$ and $N_{r,t} \sim Po(\lambda_r)$. Shocks $Z_t$ are normally distributed and operated with the standard deviation of the rate of return $\sigma_r$.

The standard Vasicek model does not simulate interest rate changes due to large financial crises. Thus we augment the model by a Poisson distributed random variable $N_{r,t}$, where $\lambda_r$ can be interpreted as the probability of a "rare disaster" (e.g. a financial crisis) and $J_r$ is a jump parameter determining how strong the rate of returns reacts to rare disasters (Huynh, Lai, & Soumaré, 2008, S. 132).

### 3.2.2 Gross Domestic Product

For modeling stochastic GDP we use the so called unobserved-components (UC) model described by Morley, Nelson and Zivot (2003).[8] According to this, the log of real GDP $\ln \mathrm{Y}_t$ can be decomposed in a trend $p_t$ and a cycle $m_t$, specified as follows

$$\ln \mathrm{Y}_t = p_t + m_t$$

$$p_t = p_{t-1} + \mu_\mathrm{p} + \eta_\mathrm{p} \quad \eta_\mathrm{p} \sim \mathrm{i.i.d.}\ N\left(0, \sigma_\eta^2\right) \tag{9}$$

$$m_t = \phi_1 \cdot m_{t-1} + \phi_2 \cdot m_{t-2} + \varepsilon_\mathrm{m} \quad \varepsilon_\mathrm{m} \sim \mathrm{i.i.d.}\ N(0, \sigma_\varepsilon^2)$$

The trend component of GDP is assumed to follow as a random walk with drift where $\mu_\mathrm{p}$ is potential output growth. The cyclical component $m_t$ is assumed to be an AR(2) process.

In the calibration section we describe the estimation procedure of the parameters of the models.

### 3.2.3 Wages and (Un-)Employment

In the simulation we assume that GDP growth affects the working population through unemployment. We estimate the elasticity, i.e. the percentage change of the unemployment rate resulting from a change in the growth rate (the so called Okun's Law) by a simple first-difference equation (Chamberlin, 2011):

$$\Delta UR_t = \beta_0 + \beta_1 g_t^\mathrm{Y} \tag{10}$$

where $\Delta UR_t$ stands for the absolute change of the unemployment rate from period *t* to *t-1* and $g_t^\mathrm{Y}$ is the growth rate of real GDP *Y*. The coefficient $\beta_1$ gives the elasticity.

[8] To be precise, we use the model which they call the UC-0-model, cf. Morley, Nelson and Zivot (2003), p.236, implying that trend and cycle innovations are uncorrelated.

The working population $L_t$ can be determined using the simulated time series of unemployment $UR_t$ together with the projected labor force $LF_t$ from the population forecast:

$$L_t = (1 - UR_t)LF_t \tag{11}$$

To model the interaction between GDP growth and gross wages we employ simple theory of the firm assuming a Cobb-Douglas production technology of the type, $Y = K^{\vartheta}L^{1-\vartheta}$ with GDP $Y$ being produced with capital $K$ and labor $L$. Assuming perfect competition on the firm side implies wages according to the marginal products. Average labor income is thus given with

$$w_t = (1 - \vartheta)\frac{Y_t}{L_t} \tag{12}$$

Assuming a constant labor income share of $(1 - \vartheta)$ the wages in the economy fluctuate with $Y_t$ and $L_t$.

## 3.3 German Social Security System

The German social security system consists of three pillars, whereby the PAYG benefits are the dominant fraction of total benefits. Since 2001 the government started to subsidize privately funded pensions as a second pillar of which the "Riester Pension plan" is most important.[9]

In the following, the two pillars are described in more detail. Note, that although we describe the benefit formula that determines $b_t$, we deviate from this formula in Section 5.2 where $b_t$ is endogenously determined in our simulations via the optimal fraction in our portfolio.

### 3.3.1 PAYGO Pension Benefits

When computing the optimal ratio of total retirement income we take the evolution of PAYG benefits as being determined by the current system. The benefit formula in Germany consists of earning points ("Entgeltpunkte") reflecting the individual working income relative to average earnings and the current pension value vt which is the monthly benefit paid per earning point.

An employee receives one earning point per year if her salary was exactly the average and more (less) than one earning point if her earnings were above (lower than) the average. We assume a benchmark pensioner (so called "Eckrentner") who earned the average gross wages throughout

[9] A third pillar, company pension schemes, will not be considered in this paper.

her 45 years of working life implying that she has accumulated $\varepsilon = 45$ earning points. The current pension value is defined as

$$v_t = v_{t-1} \cdot \frac{w_{t-1} \cdot (1 - \tau_{t-1} - a_{t-1})}{w_{t-2} \cdot (1 - \tau_{t-2} - a_{t-2})} \cdot \left( \left( 1 - \frac{RR_{t-1}}{RR_{t-2}} \right) \cdot \varphi + 1 \right). \tag{13}$$

The current pension value intends to tie the benefits to wage growth and it increases if gross wages increase. However, there are three additional adjustments made to the pension value: First, if the contribution rate $\tau_{t-1}$ rises, this burden should be shared by the retirees through lowering the level of benefits. Second, the so called "Riester factor" intends to decrease the fraction of PAYGO benefits by the simultaneous increase in privately financed funded pensions, reflected in the privately financed fraction of pensions $a_{t-1}$. The legislator introduced a certain fraction as an objective increasing up to 4% of gross wage income in 2012, which is the parameter $a_t$ used thereafter. Third, the pension value is linked to demographic change by the last term in the brackets. This sustainability factor ("Nachhaltigkeitsfaktor") links the pension value to the change of the ratio of pensioners to workers, $RR_t$. $\varphi$ is a sensitivity parameter which is fixed at 0.25 by law. This term implies that a higher ratio of pensioners to workers dampens the increases of the current pension value and thereby of average pensions. A parameter $\varphi = 0.25$ implies that roughly 25% of this increase is borne by pensioners in the form of lower benefits and 75% by workers by increased contribution rates. Overall, individual benefits from the PAYGO system are given by:[10]

$$b_t = \varepsilon \cdot v_t. \tag{14}$$

We apply equation (14) to get the results presented in section 5.3 where the PAYGO system is assumed as given.[11]

In order to calculate the contribution rate for given $b_t$ we need to calculate both sides of the budget constraint of the pension system defined as:[12]

$$C_t^w + C_t^{UI} + G_t^g + G_t^a = B_t + H_t + R_t + A_t \tag{15}$$

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[10] Note that we abstract from a safeguard clause according to § 68a SGB VI which prevents pensions to fall by law.

[11] Note that we cannot apply equation (14) for the results of section 5.2 since we precisely aim to find the optimal level of benefits from a portfolio point of view.

[12] We use capital letters for aggregate values and lower-case letters for values at the individual level.

The largest part of revenues on the left hand side are total contributions of the workers

$$C_t^w = \tau_t \cdot w_t \cdot L_t, \tag{16}$$

where Lt are people in paid work. In addition, the unemployment insurance pays contributions for the unemployed $C_t^{UI}$ and the federal government pays general $G_t^g$ and the additional $G_t^a$ grants linked to gross wages. The details of these additional income streams are relegated to the appendix.

Expenditures of the social security system are dominantly total benefits to the retirees $B_t$. These are defined as individual benefits $b_t$ multiplied by the number of retirees: $B_t = b_t \cdot R_t$. We assume constant earning points for each cohort and abstract from times of unemployment.[13] Moreover, the social security system pays for health insurance of the retirees $H_t$. Additional expenditures are payments for rehabilitation measures $R_t$ and administrative costs $A_t$ which we take into account in our simulations.

### 3.3.2 Riester Pension Plan

For the subsidized privately funded pension scheme ("Riester pension plan") various financial market products are available. Details are described in the calibration section. To get subsidies from the government a Riester pension scheme generally has to be arranged as an annuity payment that starts payouts at retirement entry after paying monthly contributions to the insurer during working life. In addition, if at least 4% of gross wages are saved (from 2012 onwards), the agent gets a 154 Euro subsidy per year and an additional lump-sum transfer of 185 Euro per child which is increased to 300 Euro for children born after 2008.[14] Additionally, we consider administration costs which are quite high for Riester products. Studies indicate administration costs of major insurance companies of around 7 to 13 percent of total contributions, cf. Wystup, Detering and Weber (2009).

We assume privately funded pensions as being only "Riester pensions". During working life each agent saves a constant fraction $s$ of gross wages so that the amount of savings each period is defined as $s_t = \gamma \cdot w_t$. Savings yield interest payments according to the stochastic rate of return $r_t$ specified in equation (8). Thus, the total amount of accumulated saving depend on

[13] Considering a constant unemployment rate of 10 percent would reduce average earning points by less than one, so we stick to the average of 45 accumulated earning points.

[14] In addition private savings less subsidies are tax deductible which is not considered in our simulation.

the series of business cycles and e.g. how many "rare disasters" occur during lifetime. We specify total savings of cohort *k* as:

$$S_k = \begin{cases} \sum_{t=t_0}^{t_0+ch} \left(s_{k,t} + x_t^{ch}\right) \prod_{i=t}^{t_0+ch} (1+r_i) \\ \sum_{t=t_0+ch+1}^{t_r-1} \left(s_{k,t} + x_t\right) \prod_{i=t}^{t_r-1} (1+r_i) \end{cases} \tag{17}$$

where $t_0$ is the first working period, $t_r$ is the first year of retirement and *ch* are the years the average agent gets subsidies for children. Here, $x_t^{ch}$ is the subsidy for an agent with children paid for *ch* years and $x_t$ is the government subsidy paid thereafter.

Riester pensions are paid as annuities. For each cohort the payments are given by: [15]

$$f_k = \frac{S_k}{LE_k^{t_r}} \tag{18}$$

where $LE_k^{t_r}$ is life expectancy of a retiree of cohort *k* at age $t_r$. For simplicity we assume that after retirement, there are no further interests paid on assets that are not yet paid out. In addition, we assume time-constant benefits and no further savings during retirement.

## 3.4 Computing the Implicit Rate of Return

The implicit return is the rate of return where the difference between the present value of benefits and contributions is just zero. The implicit rate of return of the PAYG system $i^P$ solves the following equation:

$$\sum_{t=t_r}^{T} \frac{b_t}{(1+i^P)^{t-t_r}} - \sum_{t=t_0}^{t_r-1} \frac{\tau_t \cdot w_t}{(1+i^P)^{t-t_0}} = 0 \tag{19}$$

The implicit rate of return of a fully funded system $i^F$ is analogously defined by

$$\sum_{t=t_r}^{T} \frac{f_t}{(1+i^F)^{t-t_r}} - \sum_{t=t_0}^{t_r-1} \frac{s_{k,t}}{(1+i^F)^{t-t_0}} = 0 \tag{20}$$

[15] Payouts from the fully funded system are calculated as smoothed payouts, i.e. an average private benefit during retirement.

# 4 Calibration

For the results we focus on the cohort born in 1979. We assume that the first working period of the agent is $t_0 = 22$, i.e. in 2002, where the agent becomes 1.5 children. The first year of retirement is $t_r = 67$ which is the assumed to be the standard retirement age by 2047. We choose the 1979 cohort because in the simulation the agent starts to buy a Riester pension plan in $t_0$ (which started in 2002 in Germany). We set the terminal year of life at $T = 88$ according to the life tables, cf. Federal Bureau of Statistics (2009b). Thus, we simulate the economy from 2011 until 2068.

## 4.1 Population Forecast

We use the population forecast by Börsch-Supan and Wilke (2007) from the Munich Center for the Economics of Aging (MEA) which modifies the official forecast of the Federal Bureau of Statistics (2009a) by assuming a rise in female labor participation in the future.[16]

The general assumptions are as follows: the fertility rate will stay at 1.4 children per women and life expectancy for a newborn will grow to 85.0 (89.2) for women (men) in 2060. Immigration will stay at 100.000 per year from 2014-2060. With these assumptions total population will fall from 81.5 Million in 2010 to 64.7 Million in 2060.[17]

In addition, labor participation of women within the next decades is assumed to reach the observed rates of Scandinavian countries, e.g. in Denmark, where it is as high as male participation. Also, labor force participation at older ages is projected to rise in the future. As shown in Figure A.1 in the appendix, the modified projections from MEA lead to similar projections than the status-quo scenario from the Federal Bureau of Statistics, albeit the old-age-dependency ratio does not rise as much.

## 4.2 Riester Rate of Return

Signing a Riester contract gives people the choice of various saving products. We calculate an average rate of return of Riester products by choosing a weighted average of the products that were bought between its introduction 2002 and 2010.

[16] We thank MEA for providing us with the data.
[17] The MEA-projections are only until 2060. We extrapolate the data to 2068 by cubic-spline methods.

The main type of investments are broadly classified in four categories: bank savings contracts ("*Banksparpläne*"), mutual fund savings plans ("*Fondssparpläne*"), the unit-linked pension insurance schemes ("*klassische Rentenversicherung*") and the financing for owner occupied property ("*Wohn-Riester*"), cf. Finanztest (2011) and Börsch-Supan, Gasche and Ziegelmeyer (2010). Bank savings contracts give variable rates on your bank savings reacting mostly little to the capital market. Mutual fund savings consist of funds and bonds while the unit-linked pension insurance schemes pay out a pension benefit after retirement that can vary with the profit participation depending on the economic situation. Housing-saving consists of a contract with a fixed rate of return that allows a low-interest loan when buying a house. According to the German Association of Investment and Asset Management sold Riester contracts rose from 3.3 million to 14.6 million in 2011 (BVI, 2011a). In addition, the share of products changed by an increased importance of mutual fund savings. In 2001 about 90 percent of the signed contracts were unit-linked pension insurance schemes falling to around 70 percent in 2011. Correspondingly, mutual funds increased. Bank savings stayed relatively constant at 4.7 percent while house-savings rose after the introduction in 2008 to about 3.7 percent.[18]

With the shares of Riester products we calculate a Riester portfolio by weighing the rates of return of each category to calculate a Riester rate of return $r$, cf. the approach in Börsch-Supan, Gasche and Ziegelmeyer (2010). Results are presented in Table 1.[19]

Table 1: Average rate of return of Riester products

| | Riester rate of return (Total) | Pension Insurance[1)] | Bank Savings[2)] | Mutual Funds[3)] |
|---|---|---|---|---|
| Mean | 4.89% | 4.79% | 2.50% | 4.30% |
| Median | 5.12% | 4.82% | 2.59% | 8.28% |
| Minimum | 3.87% | 4.27% | 1.97% | −9.11% |
| Maximum | 5.34% | 5.18% | 3.08% | 9.72% |
| S.D. | 0.0055 | 0.0029 | 0.0035 | 0.0764 |

*Notes:* S.D.: Standard deviation. Own calculations. Averages between 2002 and 2011. Values during the years of crisis 2008/2009 are not considered to avoid biases in our simulation. [1)] The return of pension insurance schemes is approximated by the net rate of return of life-insurances from 1980 to 2010 (GDV, 2011, S. 29; GDV, 2004, S. 26). [2)] The effective interest rate of bank savings is calculated from deposits of households with durations of two years and above, cf. Bundesbank (2011c). [3)] For the rate of return for mutual funds we take the average returns of investments into stocks, corporate bonds and sovereign bonds from the four most important home countries of securities hold by German mutual funds investors which are: Germany itself (41 percent), the US (8 percent), France (7.9 percent) and UK (6.3 percent), cf. Bundesbank (2011e). We establish four national weighted returns consisting of the three asset classes' returns.

[18] Due to data limitations we will not consider housing-saving in the simulation.
[19] Details of the rate of return calculation for mutual funds are relegated to the Appendix.

The overall Riester rate of return $r$ from the weighted average of each of the above described Riester products rose from 3.87% to 5.23% between 2002 and 2007, resulting inter alia through the increased mutual funds share. During the financial crisis 2008/09 the return fell to 1.39/2.04 percent and rose up to 5.08% since then. On average the rate of return was around 4.89 percent.

In addition to the rate of return on private savings into Riester accounts, the agents get yearly lump-sum subsidies from the government. To calculate the government subsidy we reach at an average yearly gross subsidy of 265.5 Euro per Person with children assuming an average fertility rate of 1.5 children. It is assumed that this value is paid for the first 25 years of working life, i.e. the child is born in the first working period. For the rest of the working life, the normal subsidy of 85 Euro is paid. In addition we assume administration costs by the private insurance companies amounting to half of total government contribution. This results in a yearly subsidy of $x_t^{ch} = 133.0$ when raising children and $x_t = 42.5$ for the time without children.

## 4.3 Stochastic Processes

The results of the last section are used to simulate a stochastic process in order to forecast the Riester rate of return. In addition a forecast for GDP growth will be designed which influences unemployment and gross wages and thus the social security system

### 4.3.1 Rate of Return

The stochastic process for the Riester rate of return is calculated with equation (8). We take the yearly average return of $\mu_r = 4.9\%$ as the mean, where the return during the crisis 2008-09 is not considered. This is also done when calculating volatility leading to a value of $\sigma_r = 0.01$ for the standard deviation. The amplitude $J_r$ of rare desasters is determined by means of the experiences during the current crisis. Our data reveal, that the return during the crisis was 3.5 percentage points lower than the average return. This is in line with Börsch-Supan, Gasche and Ziegelmeyer (2010), who estimate the loss of the rate of return of retirement assets during the crisis to be approximately 3 percent.

To calibrate the speed of convergence $\theta_r$, a closer look at the Riester returns is adjuvant. Due to the crisis, the return fell from 5.23% in 2007 to 1.39% in 2008. Two years later, the return was 5.08% and thus reaching the level of 2007 again. Calculations show that given $\mu_r$ and $J_r$ a value of $\theta_r = 0.5$ creates a similar convergence speed of about three years. Hence, we use this factor in our simulation.

We determine the probability of a financial crisis $\lambda_r$, i.e. of the Poisson-distributed random variable $N_r$ in equation (8), by concentrating on stock market crashes. We assume that by substitution effects such a crash will also lead to a fall of the rate of return of all other Riester products. Following Barro und Ursúa (2009) we define a crisis as a drop of the yearly return of 25 percent and more. For the DAX this results in six crises since 1960, cf. Figure A.2 in the Appendix.

With this method we can detect six crashes within 50 years leading to a probability of a stock market crash of $\lambda_r = 0.12$ per year.

### 4.3.2 GDP Growth

We estimate the parameters $\Gamma = [\mu_{\mathrm{p}}, \sigma_\eta^2, \phi_1, \phi_2, \sigma_\varepsilon^2]$ from the dynamic system (9) using Maximum Likelihood estimation where the Likelihood function is given by the Kalman filter, cf. Morley, Nelson and Zivot (2003). We use real quarterly seasonally adjusted GDP data in logs for Germany from 1991-2011 taken from Eurostat.[20]

A summary of the estimated parameters for the dynamic process of GDP is given in Table A.1 in the appendix. The estimated parameters are well in line with the findings of Morley, Nelson and Zivot (2003) for US-data. Taken the parameters from past data we forecast GDP assuming the dynamic system described in equation (9).

## 4.4 Employment and Gross Wages

In order to simulate the time series for the unemployed, we estimate the elasticity of the unemployment rate with respect to output growth according to equation (10).

Studies for Germany report an elasticity between -0.3 and -0.11 (cf. Table 2) mostly with relatively low values for $R^2$. It is common usage to consider the unemployment threshold $\beta_0$ in the estimation model. The unemployment threshold indicates how much an economy has to grow at least to lower the unemployment rate. This concept is closely related to Verdoorn's law which states that a certain growth rate (which exceeds the growth due to technological progress) is necessary to increase the employment rate (Schäfer, 2005, p. 2). In the model we concentrate on workers who pay mandatory social security contributions, so we use the unemployment rate

[20] Details of the estimation procedure are available upon request.

of these employees taken from the Federal Employment Agency (Bundesagentur für Arbeit, 2011). Data on GDP growth rate are at 2005 prices. Estimates are reported in Table 2.

Our OLS estimates for 1950-2010 gives an elasticity of $\beta_1 = -0.1146$, i.e. a one percent increase of GDP growth reduces the unemployment rate at 0.11 percentage points, if growth exceed an unemployment threshold of 3.77 percent.[21] Our results are broadly in line with other studies, especially the recent study of Chamberlin (2011) so we use $-0.11$ as the elasticity in the simulation. We assume a natural rate of unemployment of 4 percent, i.e. $UR^* = 0.04$, cf. Börsch-Supan und Wilke (2007, p. 25). In the simulation the unemployment rate cannot fall beyond this value.

Table 2: Regression results of Okun's Law for Germany

| | Time | $\beta_0$ | $\beta_1$ | R² |
|---|---|---|---|---|
| Own Estimates | 1950-2010 | 0,4316 *** | −0,1146 *** | 0,31 |
| Chamberlin (2011) | 1984-2010 | 0,0400 * | −0,1115 *** | 0,21 |
| Schäfer (2005) | 1988-2003 | 0,7280 | −0,2800 | 0,39 |
| Walwei (2002) | 1980-2000 | 0,9000 | −0,3000 | 0,30 |

*Notes:* Significant level 1% (***), 5% (**) and 10%.

The elasticity allows to compute a time series for unemployment which is used to compute employment according to equation (11). This is turn is used to compute gross wages, cf. equation (12). We take the labor share $(1 - \vartheta)$ from the last period where real-time data is available, i.e. 68% in 2009 (obtained by taking the quotient of employees' income and national income), which is close to the 40-years long-term average of 69.99%.

A summary of the estimated and chosen parameters for the simulation is presented in Table A.1 in the Appendix.

# 5 Results

We first present time series of the stochastic giving time series for real GDP, the interest rate, the working population and the unemployed as well as gross wages (i.e. labor income) processes in Section 5.1. This is the basis for the results on optimal share of social security. In Section 5.2 we present results for the optimal portfolio of the PAYG and the funded system and in Section

[21] Observe that $\Delta UR_t = 0 \Leftrightarrow \beta_0 + \beta_1 g_t^Y = 0 \Leftrightarrow g_t^Y = -\beta_0/\beta_1$.

5.3 we present results for an optimal total benefit level taking the evolution of social security benefits as being determined by the current system in Germany. In Section 6 we present results of a sensitivity analysis.

## 5.1 Simulated Economy

Simulated real GDP growth is slightly above 1% on average ranging between -0.5% and 3% within a 95% confidence interval.[22] Generally, the Riester rate of return is more volatile, cf. Figure 1.

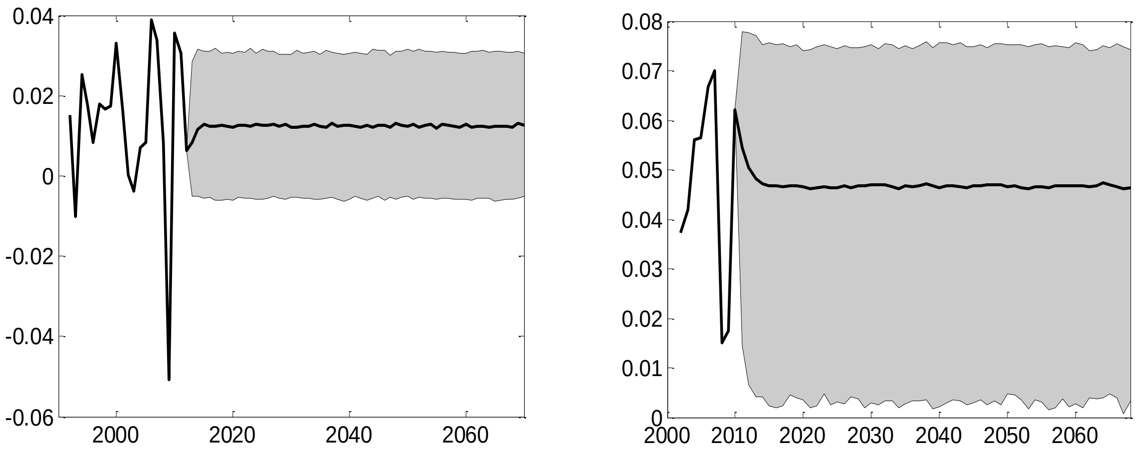


Figure 1: Real GDP growth (left panel) and Riester Rate of Return (right panel). *Notes:* The line shows real-time data until 2011 and the mean of the simulated data until 2068. The grey shaded area is the 95% confidence interval.

On average, the Riester rate of return is positive at around 5.5 percent. Here, the 95% confidence interval ranges between 0.5% and 7.5%. Note, that the return does not take on negative values. This is due to the fact that even during the financial crisis, the rate of return of the average Riester portfolio was yielding positive returns.

For the working population and gross wages we use the demographic projection from MEA.[23] The decline of the labor force leads to a decline of the working population even with an ever increasing GDP. The resulting labor shortage leads to a decline of the unemployed, cf. Figure A.3 in the Appendix. This demographic process together with increasing real GDP leads real gross wages to increase as depicted in Figure 2.

[22] Observe that the mean growth rate during the projected time span is supposed to be constant. The wiggles are due to the fact that we “only” ran 5.000 Monte Carlo simulations.
[23] Results with the status-quo scenario are not very different and are available upon request.

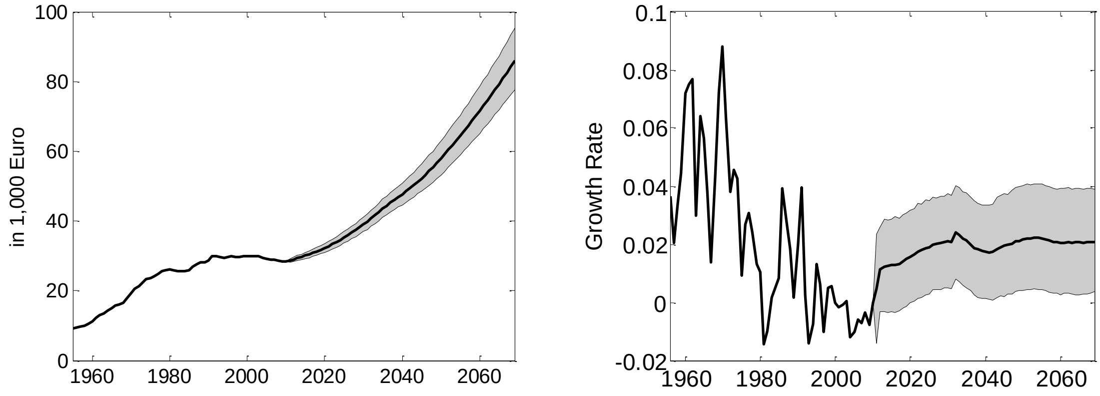


Figure 2: Real gross wages (left panel) and growth rate (right panel). *Notes*: The line shows real-time data from 1955-2010 and the mean of the simulated data until 2069. The grey shaded area is the 95% confidence interval.

The time series for real wages dripples on average during the projected time span from 2010-2069 (see left panel). This implies a growth rate of real wages of around 2 percent on average (see right panel). While this seems a rather optimistic scenario it is comparable to the historic data. Real wages in 2009 are also three times higher than in 1955. The evolution comes from growing GDP and an increased labor scarcity in the future.

The stochastics in real wages influence fluctuations within the PAYG pension scheme while the Riester rate of return represents the risk within the fully funded system. It becomes obvious that the main risk of the PAYG system comes from the demographic process.

## 5.2 Optimal Portfolio of Benefits and Private Savings

We determine the optimal share of PAYG benefits and funded pensions by assuming that total pension benefits to gross wages $b_t^{tot}$ stay constant at 70 percent. We calculate the per-period contribution rate $\tau_t$ and savings $s_{k,t}$ per cohort k for different shares $\alpha$ in order to get a total benefit $b_t + f_t$ leading to a ratio of $b_t^{tot} = 0.7$ in each period. The contribution rate $\tau_t$ is found by solving the government budget constraint.

The results show the impact of the demographic change on the social security system: for a PAYG system without a funded pillar (i.e. $\alpha = 1$) the contribution rate rises to a value of 47.7 percent of gross wages in 2069.

When calculating the privately funded pillar of social security we have to analyze cohorts. Due to our chosen data we analyze the 1979 cohort that is at age 22 in 2002 when the Riester product first began and starts saving and paying contribution at that age. The last year of retirement is then (according to the life table projections) at age 88 in 2068. If we simulate a fully funded system (i.e. $\alpha = 0$) the necessary saving rate to reach at a total benefit of 70% is around 8.2 percent on average for the 1979 cohort.

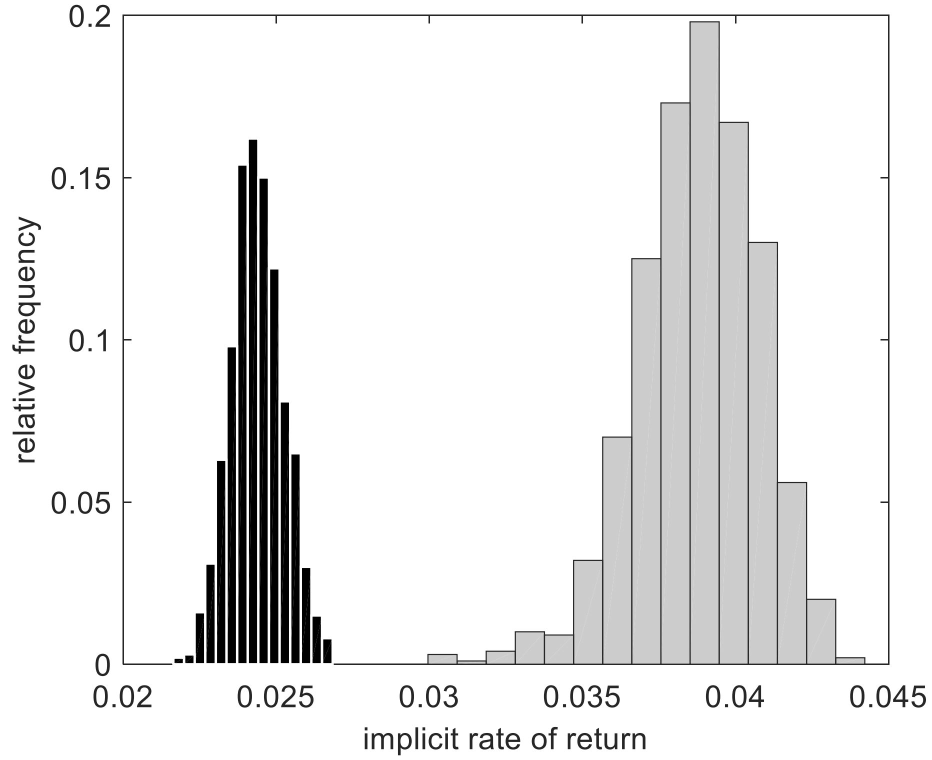


Figure 3: Distribution of rate of returns. *Notes:* Relative frequency of implicit rate of returns for the 1979 cohort, if benefits are only PAYG financed (black bars, $\alpha$ = 1) and fully funded (grey bars, $\alpha$ = 0).

Figure 3 shows the frequency distribution of the implicit returns for the 1979 cohort if benefits are paid only via the PAYG and a fully funded system respectively. Obviously, the implicit return of the fully funded system (grey bars) is higher and more volatile than the implicit return of the PAYG system. The mean lifetime implicit rate of return for the pure PAYG is 2.43 percent with a standard deviation of 0.0009. On the other hand, the implicit return for a fully funded system – in our case relying only on the Riester pension scheme – is 3.88 percent with a higher standard deviation of 0.002. The internal rate of return of the PAYG system in our simulation is higher than sometimes found in the literature. For example, Schnabel (1998) finds a rate of return of roughly 1 percent for the 1970 cohort. This is due to the fact that this study assumes real wages to grow at 1 percent on average. As outlines above we believe that real wages will grow at a higher rate due to the demographic process.

As pointed out by Dutta et al. (2000) a mixture of the PAYG and the fundes system is the better the lower the correlation of the returns of two assets. In our simulations, the correlation coefficient between the internal rate of return of a fully funded and a pure PAYG system is positive with a value of $\rho_{P,F} = 0.26$. This rather strong positive correlation still allows for partial hedging by combining the assets.

Figure 4 shows all combinations of the mean rate of return and the corresponding volatility for each portfolio. For most of the part, a higher implicit rate of return is bought with a higher volatility but this is not the case for the endpoint. The results show that a fully funded system does indeed lead to the highest implicit return.

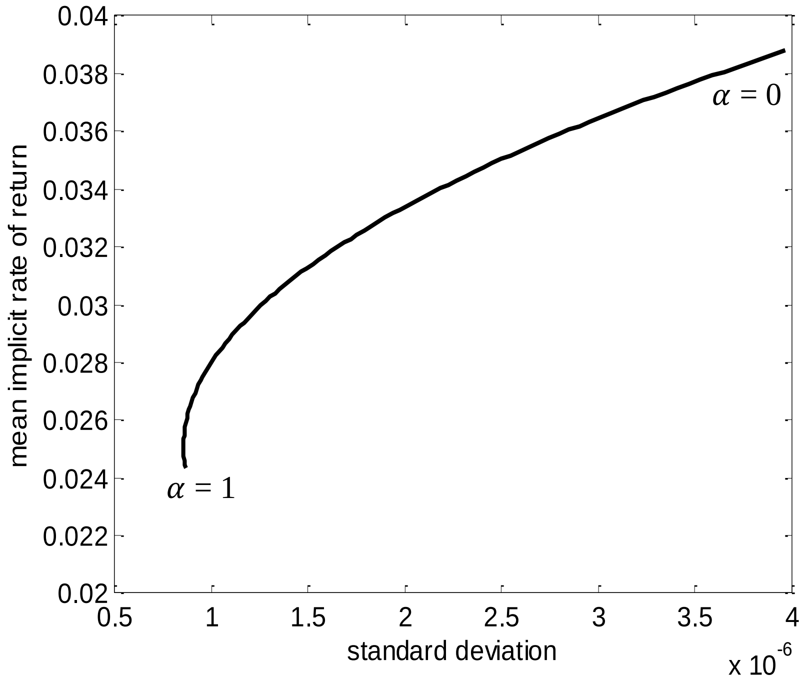


Figure 4: Relation between mean and standard deviation of lifetime IRR. *Notes:* The line shows for each fraction α the maximal implicit return and the corresponding standard deviation. A fraction α=1 implies a pure PAYG system and α=0 is a pure fully funded system.

In contrast, the pure PAYG system does not imply the lowest volatility. One can raise the mean and lower the variance by allowing a share of funded benefits of 5 percent. Thus, according to the minimal-variance-maximal-return approach the PAYG benefits should be a share of 95 percent leading to an implicit return of 2.5 percent. Of course, this approach does not explicitly takes individuals' risk-aversion into account and it does not account for the financing side of the two systems.

Table 3 summarizes the main results for the cohort born in 1979 by showing averages for the simulation period. The results confirm that there is a strong argument for a funded part in addition to the PAYG system from a portfolio point of view. The maximal implicit rate of return of 3.88% is gained by a fully funded system implying a standard deviation of 0.00199. The average necessary saving rate for retirement during working life is 8.1%. Even for a household who wishes to minimize risk there is room for funded fraction amounting to 5 percent. However, due to demographic change, the average contribution rate necessary to finance a total benefit level of 70% of wages is 36.7%.

Table 3: Main Results for Optimal Portfolio

| | | PAYG fraction α | Implicit rate of return | Standard Deviation | Contribution rate τ | Saving rate for Riester Plan |
|---|---|---|---|---|---|---|
| Max impl. rate of return | | 0.0 | 3.88% | 0.00199 | 0.0 | 8.1% |
| Mean-variance-approach | | 0.95 | 2.50% | 0.00092 | 36.7% | 0.4% |
| Max Utility | $\phi = 1$ | 0.02 | 3.85% | 0.00196 | 0.3% | 7.9% |
| | $\phi = 2$ | 0.11 | 3.72% | 0.00181 | 0.7% | 7.2% |
| | $\phi = 3$ | 0.15 | 3.66% | 0.00175 | 1.9% | 6.9% |
| | $\phi = 4$ | 0.16 | 3.65% | 0.00119 | 2.3% | 6.8% |
| | $\phi = 10$ | 0.55 | 3.08% | 0.00110 | 19.5% | 3.6% |

*Notes:* Optimal fraction α of PAYG benefits and corresponding implicit rate of return and standard deviation of the optimal portfolio between PAYG benefits and private savings invested in Riester plan. The contribution rate and the saving rate is shown as the average 2010-2069 for the 1979-cohort. Results assume a constant total benefit level of 70%.

Using the concept of lifetime utility with a per-period CRRA utility function we analyze optima depending on the degree of risk aversion. In addition, this concept takes the whole life-cycle into account including the tradeoff between higher contributions and savings during working life in order to finance the pension benefits at retirement. The results show an increasing optimal share of the funded pillar with higher degrees of risk aversion. For reasonable parameters of risk aversion (between 1 and 4), the optimal fraction ranges from 2% to 16%. Only for very high values the PAYG fraction exceeds values above one half. One reason for this outcome is the strong negative impact of demographic change on the contribution rates of the PAYG system which not only affects the internal rate of return but also decreases net income during working life.

We want to stress that the results above are calculated for a specific cohort born in 1979. This cohort is entering the labor market by the time of the introduction of Riester pensions in 2001. The agents under study have their full working life ahead where they can privately save for retirement. This is the underlying assumptions of our presented results. To highlight the importance of the cohort chosen, we redo our analysis for a cohort born in 1950. As the main difference, this cohort has only a few additional years to privately save in our simulation. Hence, when calculating an optimal portfolio their saving-possibilities are limited. Computing the optimal portfolio by maximizing lifetime utility for the cohort born in 1950 yields an optimal portfolio that only consist of PAYG benefit without any private savings, i.e. α=1, irrespective of the value of risk aversion. Hence, even if the agent exhibit only low risk-aversion, the low remaining possibilities to privately save leaves the agent to favor a full PAYG system.

## 5.3 Optimal Pension Benefit Level

In the proceeding analysis we assumed the benefit level as given by 70%. In this section, we now determine the optimal total benefit level $b_t^{tot}$ and the associated savings taking the benefits as given by the benefit formula currently in place in Germany. The optimal fraction of funded pensions, $\bar{\alpha}$, is found by maximizing equation (5).

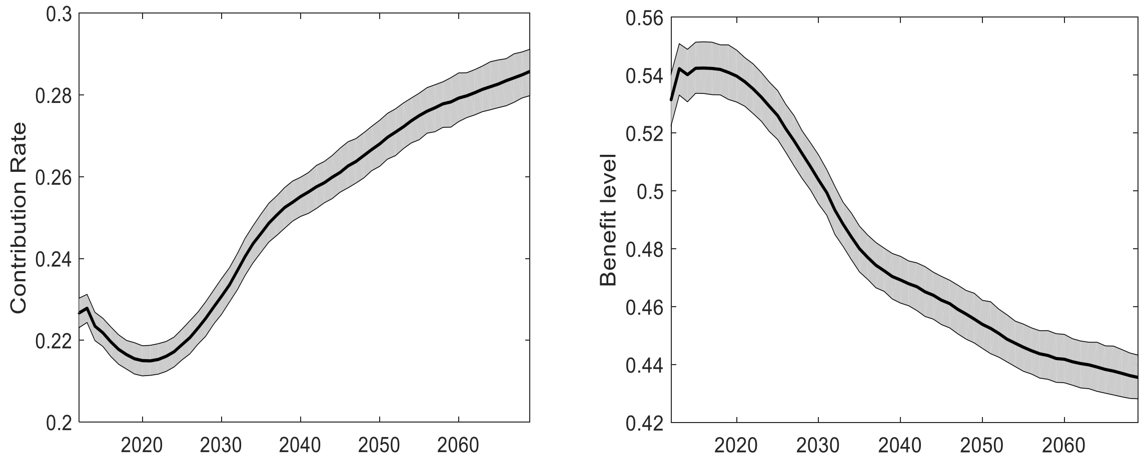


Figure 5: Simulated contribution rate (left panel) and benefit level (right panel) until 2068. The grey shaded area is the 95% confidence interval.

Figure 5 depicts the simulated profile of the social security benefits, $b_t/w_t$, and the corresponding contribution rate $\tau_t$ that balances the social security budget, where the grey area covers the 95-confidence interval. Overall, the benefits are projected to decrease to 44 percent by 2068 with a corresponding contribution rate of 29 percent.[24]

In Table 4 we present the results of the optimal total benefit level. The results from this exercise somewhat put into perspective our earlier results from the last subsection. Although, we find that even highly risk-averse agents (risk aversion larger 4) favor a significant fraction of private

[24] The confidence intervals are quite narrow since they only consider the stochastics of our processes of GDP and wages. We do not account for uncertainties in the projection of the demographic process. See the next Section for a sensitivity analysis.

savings given a certain total benefit level (cf. Table 3), this finding is altered, once we allow the agents for determining the optimal size of the total benefit level itself. Here, highly risk-averse agents are willing to give up a significant fraction of retirement income in order to avoid the risks associated with private savings. For reasonable values for the parameter of risk-aversion of 3, households favor a benefit level of 78% of average gross earnings (of which 34 percent are financed via private savings) over a 100%-benefit level which would imply higher risky savings. The implied saving rate is 3.9 percent.

Table 4: Main Results for Optimal Benefit Level

| | | Benefit Level, $b^{tot}$ | Fraction of private Savings, $\bar{\alpha}$ | Implicit rate of return | Standard Deviation | Saving rate |
|---|---|---|---|---|---|---|
| Max impl. rate of return | | 1.0 | 0.66 | 3.75% | 0.00144 | 6.5% |
| Mean-variance-approach | | 0.44 | 0.0 | 2.50% | 0.00079 | 0.0% |
| Max Utility | $\phi = 1$ | 1.0 | 0.66 | 3.75% | 0. 00144 | 6.5% |
| | $\phi = 2$ | 1.0 | 0.66 | 3.75% | 0. 00144 | 6.5% |
| | $\phi = 3$ | 0.78 | 0.34 | 3.26% | 0. 00115 | 3.9% |
| | $\phi = 4$ | 0.66 | 0.22 | 2.91% | 0. 00099 | 2.5% |
| | $\phi = 10$ | 0.50 | 0.06 | 2.13% | 0. 00084 | 0.6% |

*Notes:* Optimal total benefit level and corresponding implicit rate of return and standard deviation of the optimal portfolio between PAYG benefits and private savings invested in Riester plan. The contribution rate and the saving rate is shown as the average 2010-2069 for the 1979-cohort. Results assume that PAYG benefits follow the current law, cf. equation (5).

# 6 Sensitivity Analysis

In this section we present results from various sensitivity analysis. First, we compare our previous results with the possibility to only buy stocks instead of the (average) Riester product. This yields a higher rate of return but also a higher volatility and is not subsidized by the government. Second, we show that the assumptions about the evolution of real gross wages are quantitatively important for our results. However, assuming a more conservative wage growth path even strengthen the argument for hedging. Third, we show that changing various other assumptions of the simulation approach have only minor effects on the results.

## 6.1 Buy Riester Products or Invest in Stocks?

Riester products are relatively well protected by law. The nominal payments into the system must be guaranteed by the insurer. As described above more than two thirds of the products chosen are classical pension insurances which yield relatively low rate of returns but are also relatively riskless.

In this section we analyze the effects of investing into stocks rather than buying an average and relatively save and subsidized Riester product. To this end, we assume that the agent directly invests in the stock market assuming an average annual return of $\mu_r = 8\%$ with a standard deviation of $\sigma_r = 0.08$. Hence, we assume that the rate of return increases by roughly 3% compared to the Riester product used in our main simulation – but at the same time the standard deviation rises by a factor of 8. The idea is that from a portfolio point of view it might be better to buy a higher return asset which is more risky and hedge this with the PAYG pension benefit. The resulting portfolio might yield better results.

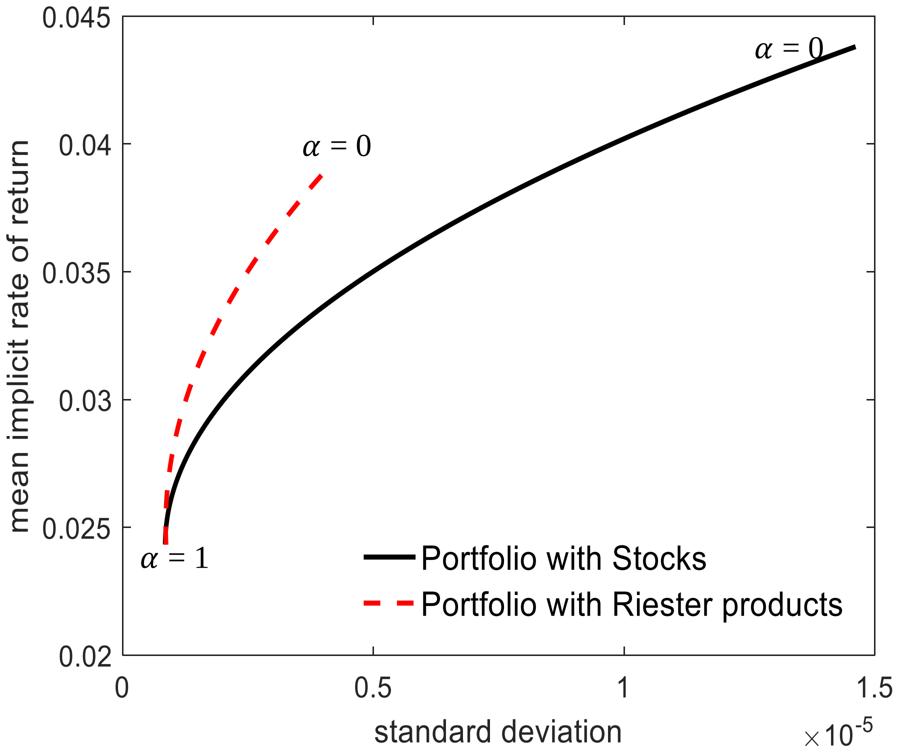


Figure 6: Lifetime IRR for two different portfolios. *Notes:* The line shows for each fraction α the maximal implicit return and the corresponding standard deviation. A fraction α=1 implies a pure PAYG system and α=0 is a pure fully funded system. The red dashed line depicts the portfolio with stocks (mean return 8% with a SD of 0.05), the black line shows the portfolio with Riester products (mean return 5.5% with SD of 0.01).

However, it turns out that with the safer Riester savings it is possible to generate portfolios yielding the same rate of return with lower risk. Figure 6 shows all combinations of the mean rate of return and the volatility, where private savings are invested in the stock market (red dashed line), compared to the Riester product used for the main results (black line). The figure shows that a portfolio with Riester plans yield higher returns for a given standard deviation up to the maximal rate of return that is possible with this portfolio. Only if the agent is willing to take up more risk this new portfolio can result in higher returns.

## 6.2 Flat Real Wage Profile

We assume rising real wages throughout our simulation. By the end of the projection in 2068 real gross wages are roughly three times larger than in 2010. We take the view that the low – or even declining – growth rate of real wages during the last 20 years was an exceptional case due to German reunification. Rather, real wages will grow again as they did until 1991. Since the growth rate of real wages is a major determinant of the rate of return of the PAYG system this is a crucial assumption. In this subsection we model the time series of real wages more conservatively. We calculate wages employing the elasticity of GDP with respect to wages using the estimated elasticity $\beta_1$ which determines the working population. Assuming perfect competition and wages paid according to their marginal products the elasticity of labor income with respect to GDP can be calculated with

$$\varepsilon_{w,Y} = \frac{\partial w_t}{\partial Y_t}\frac{Y_t}{w_t} = \left[1 + \beta_1 \frac{1+g_t^Y}{1-UR_t}\right] \tag{21}$$

where the coefficients $\beta_1$ are given from the estimation results in equation (10).[25]

The resulting time series of real wages is rather flat with a mean growth rate of 0.01 percent. This leads to overall lower implicit returns for both systems. But since the volatility of wages is also almost non-existent in this setting, the implicit rates of returns from a PAYG and a fully funded system are more different from a portfolio point of view, cf. Figure 7.

The mean rate of return achieved with a fully funded system is 2.3%. The portfolio with minimal variance under maximal return consists of a pure PAYG system implying an implicit return of 0.83%. Applying the utility concept reveals rather low fractions for the PAYG system even for high parameters of risk aversion. The utility maximizing fraction of PAYG benefits with a risk aversion parameter of $\emptyset = 10$ is only 20% compared to 59 percent in the baseline simulation, cf. Table 3.

[25] See appendix for a derivation of equation (21). Estimation results are available upon request.

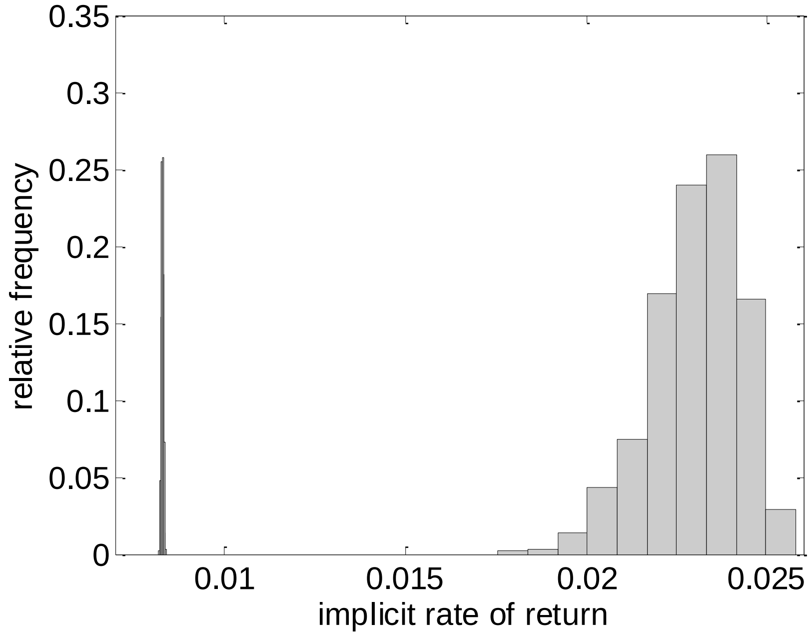


Figure 7: Relative frequency of implicit rate of returns for the 1979 cohort with different simulation of wages, if benefits are only PAYG financed (black bars, $\alpha$ = 1) and fully funded (grey bars, $\alpha$ = 0).

## 6.3 Further Robustness Checks

In this subsection we briefly show that changing the assumed demographic process, and a model without rare disasters only have minor effects on the general results.[26]

### 6.3.1 Alternative Demographic Projections

As outlined in the calibration we use a demographic projection which is modified from the official projections by MEA. Using official data instead implies an even further decline of the working population and a corresponding rise in retirees. The effects on the results are as expected, but not large: the implicit return of a pure PAYG system declines from 2.4 to 2.1%. All other results are not much effected.

### 6.3.2 Are Rare Disasters Important?

We simulated the stochastic process of the rate of return on privately funded pension benefits including the possibilities of rare disasters of a magnitude comparable to the current financial crisis. Our results reveal that the impact of large drops in the rate of return for privately saved pension funds large washes out on average. This is due to the large time span of private savings

[26] Results for this subsection are available upon request from the authors.

where short breakdowns of the return do not matter that much, see Börsch-Supan, Gasche, & Ziegelmeyer (2009).

# 7 Conclusion

Our analysis sheds light on a largely overlooked argument in trying to find an optimal combination of a PAYG social security and a funded system: the advantage of hedging. A funded pension system is associated with higher and more volatile implicit rate of returns compared to a PAYG system. We analyze a combination of the two pension systems from a portfolio perspective. As our main finding, a part of retirement income should be funded because this simultaneously increases the rate of return *and* lowers the overall risk compared to a pure PAYG system. Taking risk aversion into account we find that – for reasonable parameter values – the fraction of benefits that are privately funded should be much higher from a portfolio perspective compared to what is observed in Germany.

Our results also highlight a second – well known – argument for a higher fraction of funding. Keeping the pension benefit level constant in the next decades requires very high contribution rates within a PAYG system due to demographic change. In recent years the extension of private funding as part of old-age pension payments has been justified with the burden of demographic change.

While our results suggest a higher fraction of privately funded pensions is favorable, the Riester pension was subject to increased criticism during the debate about the recent social security reform. Politicians have labeled the privately funded pillar as less robust or considered it a failure.[27] A similar sentiment seems to prevail across society; for the last three years the number of Riester pension contracts has stagnated slightly above 16 million and about every fifth contract is on hold (i.e. no contributions are currently paid). Furthermore, the share of mutual funds – a type that yields higher returns on average – has hardly ever exceeded 20 percent, cf. BMAS (2016).

In this paper we show that a funded part of the benefits is not simply motivated to maintain financial sustainability of the social security system. Rather, a combination of both systems is optimal from a portfolio point of view with higher rates of returns and lower volatility.

[27] See, for example, www.welt.de, "„Gescheitert" - Seehofer will Riester-Rente abschaffen", 8.4.2016.

# 8 Appendix

## 8.1 Population Forecast

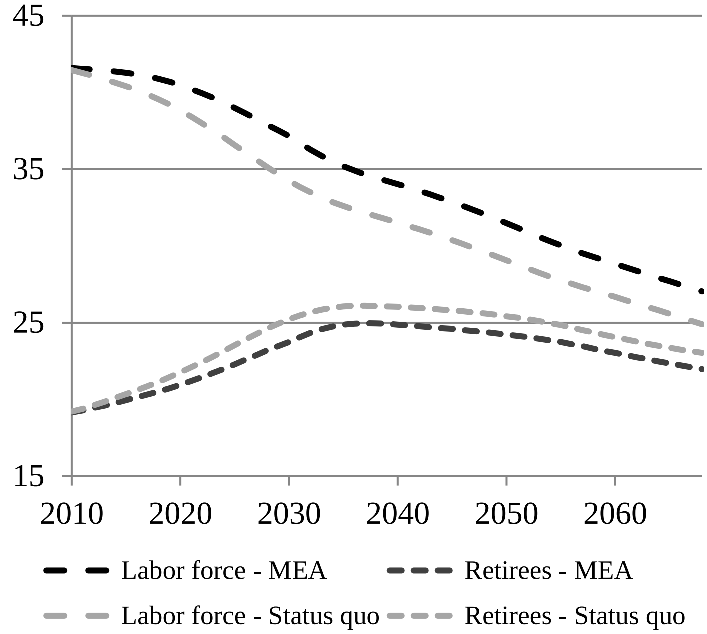


Figure 8: Projected Labor force and retirees until 2060 (in Million). *Source*: MEA. Since data is not available for the years 2061-2068 we use cubic spline extrapolation for the missing years.

## 8.2 Rate of Return

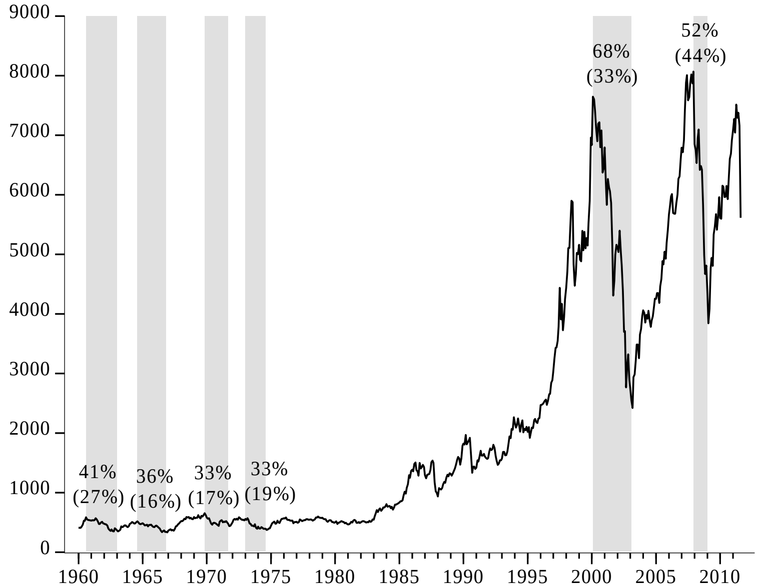


Figure 9: Stock market crashes of the DAX since 1960 (drop above 25% for one year or longer). The length of one drop is shaded grey, the numbers give the percentage drop and the average drop per year in parenthesis. Own calculations.

## 8.3 Calibrated Parameters

Table 5: Estimated and chosen parameter

| Variable | Explanation | Source |
|---|---|---|
| Timing | | |
| $t_0 = 22$ | Age at first working year in 2002 for cohort 1979 | |
| $t^r = 67$ | Retirement entry in 2047 for cohort 1979 | |
| $T = 88$ | Last year alive in 2068 for cohort 1979 | Destatis (2009b) |
| Preferences | | |
| $\beta = 0.98$ | Discount factor | |
| $\emptyset = \{0,10\}$ | Relative risk aversion | |
| Population | | |
| $\{LF_t\}$ | Labor force | MEA projections |
| $\{R_t\}$ | Retirees | MEA projections |
| Riester Rate of Re-turn | | |
| $\mu_r = 4.89\%$ | Mean rate of return | Own estimation |
| $\sigma_r = 0.01$ | Standard deviation of the Rate of return | Own estimation |
| $\theta = 0.5$ | Convergence speed | Own estimation |
| $\lambda_r = 0.12$ | Probability of a stock market crash | Own estimation |
| $J_r = -0.035$ | Amplitude of a stock market crash | Own estimation |
| $x_t^{ch} = 133.0$ | Riester subsidy for an agent with children (in €) | |
| $x_t = 42.5$ | Riester subsidy after the child has grown up (in €) | |
| GDP process | | |
| $\mu_{\mathrm{p}} = 0.3096$ | Mean of real log GDP trend | Own estimation |
| $\sigma_\eta^2 = 0.5795$ | Variance real log GDP trend | Own estimation |
| $\phi_1 = 1.5067$ | Coefficient of the first lag of cyclical component of real log GDP | Own estimation |
| $\phi_2 = -0.6199$ | Coefficient of the second lag | Own estimation |
| $\sigma_\varepsilon^2 = 0.4804$ | Variance of cyclical component of real log GDP | Own estimation |
| Unemployment and wages | | |
| $\varepsilon_{UI,Y} = \beta_1 = -0.1146$ | Elasticity of the unemployment rate w.r.t. GDP growth changes | Own estimation |
| $(1 - \vartheta) = 044.$ | (Raw) labor income share in 2009 | |
| $\{w_t\}$ | Real wages*) | RV in Zeitreihen (2010) |

*) Real wages are calculated with the HVPI for prices at 2005.

## 8.4 Simulated working population and unemployed

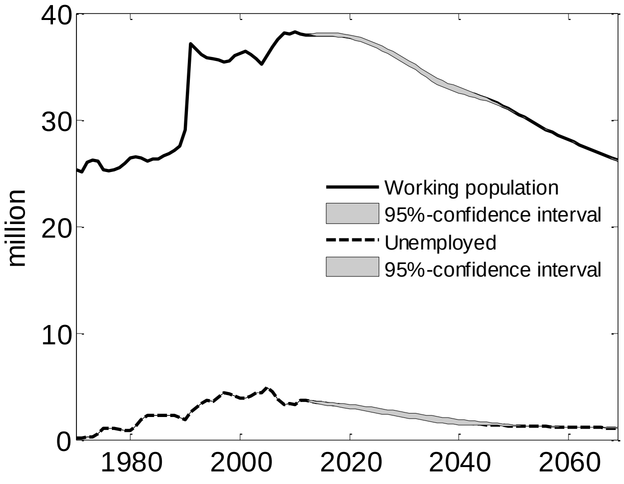


Figure 10: Working population and Unemployed. *Notes:* The line shows real-time data until 2010 and the mean of the simulated data until 2069. The grey shaded area is the 95% confidence interval.

## 8.5 Components of the social security budget constraint

Contributions $c_t^w$ of the workers in period *t* are defined as:

$$c_t^w = \tau_t \cdot w_t \cdot L_t \tag{22}$$

Where $L_t$ are people in paid work.

In addition, the unemployment insurance pays contributions for the unemployed for 80 percent of unemployment benefits. These benefit level is 60 percent of last year's earnings. It is assumed that unemployment is one year on average so that we can use last year's average net wage income $w_{t-1}^n$.[28]

The unemployment insurance thus pays 48 percent of the contribution of an average worker, leading to:

$$B_t^{AL} = \tau_t^{RV} \cdot 0{,}48 \cdot w_{t-1}^n \cdot U_t \tag{23}$$

where $U_t$ are the number of unemployed in period *t*.

[28] The higher benefit of 67 percent will not be considered here.

Federal grants can be distinguished between general federal grants $G_t^g$ and additional grants $G_t^a$. The former is linked to the change of the contributions paid by workers and is reduced yearly by 340 million Euro (cf. (§ 213 Abs. 2a S. 1 SGB VI), leading to:

$$G_t^g = G_{t-1}^g \cdot \frac{w_{t-1}}{w_{t-2}} \cdot \frac{\tau_t}{\tau_{t-1}} - 3.4 \cdot 10^8 \tag{24}$$

An additional payment $G_t^a$ to the social security system by the federal government is granted (§ 213 Abs. 3 S. 1 SGB VI) to account for changes of the sales tax rate and average gross wages multiplied with a factor $\theta_t$ leading to:[29]

$$G_t^a = G_{t-1}^a \cdot \theta_{t-1} \cdot \frac{w_{t-1}}{w_{t-2}} - 4{,}09 \cdot 10^8 \tag{25}$$

The additional grant is also reduced by 490 Million Euro yearly.

On the expenditure side, the Health insurance pays for half of the health-care contributions of the retirees with rate $\tau_t^H$. In addition it pays for rehabilitation measures. This gives:

$$H_t = 0{,}5 \cdot \tau_t^R \cdot B_t \tag{26}$$

and

$$R_t = R_{t-1} \cdot \frac{w_{t-1}}{w_{t-2}} \tag{27}$$

We assume that administrative expenditures $A_t$ depend on gross wages and rises with total retirees in the economy. We follow Wilke (2004, S. 13) and assume the following:

$$A_t = A_{t-1} \cdot \frac{w_{t-1}}{w_{t-2}} \cdot \left(1 + 0{,}1 \cdot \left(\frac{R_{t-1}}{R_{t-2}} - 1\right)\right) \tag{28}$$

Together with the overall budget constraint (15) of the social security system we get for the contribution rate:

$$\tau_t = \frac{B_t + H_t + A_t - G_t^a}{w_t \cdot L_t + 0{,}48 \cdot w_{t-1}^n \cdot U_t + \frac{G_{t-1}^a}{\tau_t} \cdot \frac{w_{t-1}}{w_{t-2}}} \tag{29}$$

[29] Since we assume a constant sales tax of 19 percent throughout the simulation the influence of changes in the tax rate drops out.

## 8.6 Calculating the mutual funds rate of return

The rate of return of mutual funds is a weighted average of equity and bond returns. As statistics reveal (cf. Bundesbank, 2011e), the bigger part of German mutual funds' assets is allocated to German stocks or bonds (41 percent), followed by investments to securities from USA (8 percent), France (7.9 percent) and UK (6.3 percent). To calculate an average return of mutual funds we distinguish between stocks, corporate bonds and government securities. We calculate the yearly rate of return for the asset classes in each country and compute a national weighted average by using the proportions from Table A.2.[30] We aggregate the national returns to an overall mutual funds return by weighting the national returns with the country shares stated above. We use national stock indices (DAX, S&P, CAC, FTSE) to obtain the return of stocks by calculating the annual return in each month and taking the average over the year. To estimate the return of corporate bonds, we employ iBoxx® corporate bonds indices. These indices are products of Markit Group Ltd., a provider of financial data, and represent the investment grade market for corporate bonds in EUR, GBP and USD. Prices for all bonds stem from several major financial institutions. Again we take the yearly average of monthly data to compute the return of corporate bonds. Since country-specific indices are not available we assume for German and French corporate bonds similar performances. For government securities we take the average over monthly returns of securities with a ten years maturity.

Table 6: Allocation of German mutual funds' assets into national securities.

| Country | Stocks | Corporate Bonds | Government securities |
|---|---|---|---|
| Germany | 61.27% | 18.75% | 19.98% |
| USA | 78.75% | 17.35% | 3.90% |
| France | 55.99% | 29.36% | 14.65% |
| UK | 62.35% | 34.86% | 2.78% |

## 8.7 Calculating the elasticity of labor income to GDP growth

The elasticity of labor income to GDP is defined as $\varepsilon_{w,Y} = \frac{\partial w_t}{\partial Y_t}\frac{Y_t}{w_t}$. We use the following equations:

[30] Note that for simplicity we assumed the proportions to be constant over time.

$$w_t = (1-\vartheta) \cdot \frac{Y_t}{L_t} \tag{30}$$

$$L_t = (1-UR_t)LF_t \tag{31}$$

$$\Delta UR_t = UR_t - UR_{t-1} = \beta_0 + \beta_1 g_t^Y = \beta_0 + \beta_1 \left(\frac{Y_t}{Y_{t-1}} - 1\right) \tag{32}$$

From these equations we can calculate the unemployment rate in period *t* with

$$UR_t = t \cdot \beta_0 + \beta_1 \sum_{i=1}^{t} g_t^Y + UR_0 \tag{33}$$

From this we can derive

$$\Rightarrow L_t = \left(1 - t \cdot \beta_0 - \beta_1 \sum_{i=1}^{t} \frac{Y_i}{Y_{i-1}} - UR_0\right) LF_t \tag{34}$$

$$\Rightarrow w_t = (1-\alpha) \cdot \frac{Y_t}{\left(1 - t \cdot \beta_0 - \beta_1 \sum_{i=1}^{t} \frac{Y_i}{Y_{i-1}} - UR_0\right) LF_t} \tag{35}$$

Thus, the elasticity can be calculated as

$$\varepsilon_{w,Y} = \frac{\partial w_t}{\partial Y_t} \frac{Y_t}{w_t} \tag{36}$$

$$\varepsilon_{w,Y} = \left[\frac{(1-\vartheta)}{L_t} + (1-\vartheta) Y_t \cdot \left(-\frac{1}{L_t^2}\right) \cdot \left(-\beta_1 \frac{1}{Y_{t-1}} LF_t\right)\right] \frac{Y_t}{w_t} \tag{37}$$

$$\varepsilon_{w,Y} = (1-\vartheta) \left[\frac{1}{L_t} + \beta_1 \frac{Y_t}{Y_{t-1}} \frac{LF_t}{L_t^2}\right] \frac{L_t}{(1-\vartheta)} \tag{38}$$

$$\varepsilon_{w,Y} = \left[1 + \beta_1 \frac{1+g_t^Y}{1-UR_t}\right] \tag{39}$$

Which is the equation given in the main text.

Real gross wages are then calculated with

$$w_t = \varepsilon_{w,Y} \cdot [g_t^Y + 1] \cdot w_{t-1} \tag{40}$$

We test our assumed stochastic process by comparing the calculated elasticity (using β_1=-0.11 throughout) with the historical data using data on labor income and GDP growth. The elasticities calculated from the data are much more volatile but the mean of 0.63 is close to the mean of the simulated elasticities of 0.68. The assumptions about the stochastic processes of GDP and the rate of return lead to reasonable results with respect to the mean elasticity of labor income with respect to GDP.

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