Combining Insurance Against Old-Age Risks to Accommodate Socioeconomic Differences in Long-Term Care Use and Mortality

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Abstract

We study adverse selection in annuity and long-term care (LTC) insurance markets and potential gains from a combination of insurances. Using unique administrative data from the Netherlands, we find opposing socioeconomic gradients in mortality and LTC which strongly differ over gender and informal care possibilities. These heterogeneous risks imply adverse selection for single annuity-, and LTC insurances. We theoretically and empirically derive conditions for an optimal combination of insurances minimizing adverse selection. Our results indicate that despite reverse gradients, a combined life care annuity is best feasible for single men and women but less so for married men and women.

Keywords: health inequalities, long-term care insurance, annuities, life-care annuity, negative externalities
JEL classification: I13, I14, G52, J14

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1 Introduction

Within the context of aging societies, the proper design of old-age insurance systems becomes increasingly salient. In private markets, a strong tendency to underinsure longevity risk and the risk of needing long-term care (LTC) has been empirically observed, often referred to as the annuity puzzle and LTC insurance puzzle, see Lambregts and Schut (2020) for a review. Adverse selection is one explanation for the limited market sizes, arising when those with above-average life expectancy more often buy annuities, and those with high expected long-term care needs more often buy LTC insurance.[1] Another explanation for the low demand for LTC insurance is the availability of informal care from the spouse or other family members (Mommaerts, 2023). To reduce adverse selection incentives, combining insurances to hedge long-term care- and mortality risks when they are negatively correlated has been proposed.[2] Despite its theoretical potential, old-age insurances that combine LTC insurance with annuities are still not very common, and its feasibility is poorly understood.[3]

This paper quantifies socioeconomic and socio-demographic differences in long-term care use and mortality and evaluates the implications for combined insurance against these risks. We theoretically derive conditions for a combined insurance that minimizes adverse selection incentives. We then quantify differences in long-term care use and mortality over lifetime income employing a multi-state model using unique Dutch administrative data of over 3 million individuals aged 65 and above. Using these results we evaluate the factors that are important for a combined life care annuity.

The point of departure for our study is the well-known socioeconomic gradient in longevity, according to which individuals with lower income die earlier than those with high incomes, cf. Deaton (2002). For long-term care use, low income individuals tend to be less healthy at older ages and require more LTC. Both of these socioeconomic

[1]cf. Finkelstein and Poterba (2004). However, the size of adverse selection problems in the LTC insurance market is subject to debates, cf. Brown and Finkelstein (2007), Brown and Finkelstein (2008), and Boyer et al. (2020), among others. Most notably, preference heterogeneity – low risks have a high preference for insurance – might even imply advantageous selection, cf. Finkelstein and McGarry (2006). We assume homogeneous preferences in this paper and take the positive correlation of private information and insurance coverage as given.

[2]Murtaugh et al. (2001), Brown and Warshawsky (2013), Webb (2009), Solomon (2022),De Donder et al. (2022).

[3]The American Association for Long-Term Care Insurance highlights the favorable experience with LTC combination products over stand-alone LTC insurance; however, the number of policies sold remains limited, see: `https://www.aaltci.org/long-term-care-insurance/learning-center/ltcfacts-2019.php` and `https://www.aaltci.org/linked-benefit-faqs/`.

gradients are significantly affected by gender and marital status. Importantly, the socioeconomic gradients and its heterogeneity greatly matter for the design of insurances. The implications apply to both private and public insurance systems.

For public social security, socioeconomic differences in mortality imply a redistribution of benefits from lower incomes, who die early, to higher incomes, who receive benefits for a more extended retirement period.[4] In private insurance markets, the implied differences in premium returns followed by inequalities in mortality can yield adverse selection problems. Pricing at average life expectancy would imply an actuarially unfair premium for higher income individuals, contributing to under-annuitization (Brown and Finkelstein, 2008). The picture is reversed for LTC insurance. Here, individuals with lower income tend to require more long-term care. This gradient imposes an opposite redistribution of benefits via LTC insurance from higher to lower income individuals in public insurance systems. In a private insurance market, medical underwriting and potentially low take-up rates of private LTC insurance might be the consequence (Braun et al., 2019). The negative correlation between longevity and long-term care needs implies that individuals with lower incomes are seen as lower risk types in the annuity market and higher risk types in the LTC insurance market, with the opposite for high income individuals. From a private insurance perspective, combining the two insurances to hedge these risks is appealing to reduce adverse selection problems.

In our study, to understand the implications of socioeconomic and socio-demographic differences in long-term care use and mortality for combined old-age insurance, we extend the standard adverse selection model by Einav et al. (2010). We formally derive an optimal combination of LTC insurance and pension annuity that minimizes adverse selection. More specifically, we obtain an expression showing how the optimal combination of insurance depends on three factors: (1) the mean duration in each of the two states 'no-long-term care use' and 'long-term care use', (2) the variances of money's worth for stand-alone LTC and annuity insurance over individual types, and (3) the correlation between the money's worth of the two insurances.

Next, we establish stylized facts on the socioeconomic gradients in longevity and LTC.

[4]For the US social security system, Groneck and Wallenius (2021) show that the (intended) progressivity turns regressive once the differences in life expectancy over socioeconomic status are considered. In a companion paper (van der Vaart et al., 2021), we quantify the welfare effects of social insurance programs stemming from inequalities in long-term care needs and mortality in a dynamic structural model.

We estimate the joint distribution of long-term care use and remaining life expectancy at age 65 by lifetime income, gender, and marital status. Exploiting rich administrative data provides us with sufficient observations also for the oldest-old, which is crucial for reliably estimating long-term care incidences. We develop a multi-state model and employ a recently developed method to estimate the underlying mixed proportional hazard rates (van der Vaart and van den Berg, 2023), incorporating frailty and allowing for time-varying covariates to capture the transition from being married to a single-person household.

We study the impact of these estimates for the design of insurances. We quantify adverse selection incentives for stand-alone pension annuity and LTC insurance measured as any deviation of the individual risk from actuarial fair pricing (i.e., non-zero premium returns). We then analyze the optimal combination of the insurances that minimizes adverse selection stemming from socioeconomic and socio-demographic inequalities. Our results allow us to understand the feasibility of combined old-age insurance for different socioeconomic groups and its determinants.

We have two main contributions. First, we establish new stylized facts simultaneously documenting a positive gradient in longevity and a negative gradient in long-term care use over lifetime income. We highlight to what extend informal care possibilities – proxied by having a spouse – affect these differences. Previous literature has studied this in isolation and focused on formal care only (cf. Kalwij et al. (2013) and Rodrigues et al. (2018), for example). Second, we theoretically and empirically study the optimal combination of insurances by determining the optimal specific benefit level for each future state of the world. Two additional factors – the relative duration and the heterogeneity in risks – are shown to be important for a combination of insurances, next to the the well-known negative correlation of risks. We further emphasize the need for group-specific premia to reduce adverse selection problems. Previous literature studied given insurance products (e.g. Murtaugh et al. (2001)) and focused on the negative correlation between risks as the precondition for a successful combination, cf. Webb (2009), Solomon (2022).

We find substantial socioeconomic inequalities in long-term care use and mortality. The difference in remaining life expectancy at age 65 between the bottom and the top lifetime income quintile is 4.0 years for men and 2.3 years for women. Women in the bottom income quintile spend an additional 1.7 years in long-term care after age 65 than those in the top income quintile, while for men, this difference is 1.1 years. Hence, gender matters for the income gradient, which is stronger for men in terms of mortality, but stronger for women in terms of long-term care. Regarding informal care possibilities, proxied by having a spouse, being married reduces long-term care duration by 22% for men and it substantially flattens the socioeconomic gradient. At the same time, this is far less pronounced for women, potentially due to the high likelihood of outliving the spouse.

The implied consequences for valuing insurances show for LTC insurance a large positive premium return of +30 percent for the lowest income quintile and a negative premium return of –17% for the highest income. The gradient of the premium returns for annuities is reversed but flatter and ranges from –9% to +4% for the lowest and the highest income quintile.

Guided by our theory we determine the optimal combination of annuity and LTC insurance. Combining both insurances reveals that this is unfeasible with a uniform premium for everyone due to large gender-differences in long-term care use and mortality and a positive correlation of premium returns over gender. Group-specific premia yield large differences for the optimal insurance products over gender and marital status. Our results suggest that a life care annuity seems feasible for single men and women but less so for married men and women, due to unfavorable variances and correlations of the risks for these groups.

Our analysis is not limited to a combination of annuities and LTC insurance but holds more general for any bundling of insurances. Bundling risks in insurances is a widespread practice ranging from life-insurance with LTC-rider to home-car insurances, see Eling and Ghavibazoo (2019) and Solomon (2022) for further examples of combining insurances. Our results can help guiding the design of such bundled insurance products and inform about its feasibility to reduce adverse selection problems.

The remainder of the paper proceeds as follows. The following Section 2 gives a brief literature review. Section 3 presents the theoretical model and Section 4 describes institutional details, the data, and the empirical approach. Section 5 presents the results, Section 6 discusses the main results and Section 7 concludes.

2 Literature

Our paper combines three related strands of literature studying (1) the causes of the annuity- and LTC insurance puzzles, (2) the potential to bundle insurances, and (3) the estimation of socioeconomic and socio-demographic differences in long-term care and mortality.

Our paper focuses on adverse selection and informal care possibilities as two factors affecting the low demand for annuities and LTC insurance. However, many other explanations for the so-called annuity- and LTC insurance puzzle have been put forward. Most notably, the risk for high out-of-pocket expenses for health-related expenses and bequest motives imply a tendency to hold sufficient liquid assets to prevent hitting the borrowing constraint, which implies low annuitization, cf. Lockwood (2018), Ameriks et al. (2018). Davidoff (2009) point to the importance of home equity, which can serve as a substitute for annuities and LTC insurance to some extent. Reichling and Smetters (2015) emphasize the role of correlated risks introduced via health shocks that simultaneously affect longevity and uninsured medical costs as a source for low valuation of annuitization. Pauly (1990) and Zweifel and Strüwe (1998), and more recently, Mommaerts (2023) and Coe et al. (2015) stress the importance of informal care availability for the low demand for private LTC insurance.

Most related to our approach are studies evaluating loads or the money's worth of insurance, which we will also apply in our analysis to evaluate adverse selection problems. Brown and Finkelstein (2007) find significant loads in the long-term care insurance market pointing to actuarial unfair pricing which varies by demographic and socioeconomic characteristics. Brown and Finkelstein (2008) determine large differences in the willingness to pay for insurance in a life-cycle setting given the current government welfare system between insurances with these loads or without. Similarly, Mitchell et al. (1999) estimate the willingness to pay for actuarially fair pricing in the annuity market using a money's worth concept.

Theoretically, the extension of the standard adverse selection model to multiple risks to compare separate versus 'umbrella' contracts has been studied by Fluet and Pannequin (1997) focusing on the relationship between partial coverage and low-risk exposure under multiple risks, Gollier and Schlesinger (1995) analyzing the optimal structure of deductibles , and Picard (2020) studying optimal risk splitting in multidimensional screening models. Webb (2009) and Solomon (2022) investigate life care annuities directly. Webb (2009) sets up an adverse selection model in the presence of preference heterogeneity and unfair pricing, showing that the bundled product can be welfare-improving. Closely related to our theoretical model is Solomon (2022), who shows that the correlation structure and whether selection is adverse or advantageous are the key elements for the welfare effects of bundling. Solomon (2022) does not analyze an optimal combination of insurances, though.

Murtaugh et al. (2001) and Brown and Warshawsky (2013) have empirically studied the attractiveness of life care annuities relative to single products by determining how a combined product can be offered with a lower premium and less strict medical underwriting to attract more people. De Donder et al. (2022) show that bundling in a life care annuity can yield advantageous selection solely assuming differences in agent's risks.

Our paper also relates to the literature studying socioeconomic differences in mortality and long-term care. Pijoan-Mas and Ríos-Rull (2014) provide age-specific estimates for the negative relationship between mortality and socioeconomic status. Kalwij et al. (2013) also estimate longevity differences over income and gender using Dutch administrative data and report similar results to what we find. Similarly, a negative relationship between long-term care needs and long-term care use and socioeconomic status has been documented (Ilinca et al., 2017; Rodrigues et al., 2018; Garcia-Gomez et al., 2019; Tenand et al., 2020). These findings align with the well-documented gender-health paradox, stating that women indeed do live longer but tend to be less healthy (Case and Paxson, 2005; Oksuzyan et al., 2008).

3 Adverse selection model with multiple risks

We extend the model of Einav et al. (2010) to describe how adverse selection for a stylized stand-alone annuity and LTC insurance can be reduced by a combined life care annuity and show that this insurance is welfare-increasing.[5] A precondition for this to work is a negative correlation between long-term care- and survival risk. We then use this simple framework to derive an optimal combination of the two insurances, allowing us to single out its determining components. We focus on comparing a world with single insurances

[5]See also Einav and Finkelstein (2023), the 'self-indulgent' survey describing the recent studies using the Einav-model.

to a world of a bundled product, which enables us to derive an optimal bundling in the sense that adverse selection problems are minimized. We abstract from multiple important aspects – such as screening, partial insurance, and the choice between stand-alone and bundled insurance – so that our simple model allows us to focus on optimally combining the two insurances and study the drivers of the optimal combination.

Suppose there is a continuum of individual types $\xi \in \Xi$ with distribution $G(\xi)$ who live for two periods. They differ by their probabilities of survival $s(\xi)$ and probability $q(\xi)$ to become in need of long-term care associated with costs of X. The probabilities are private information. Individuals receive utility U from consumption and are risk-averse with with $U' > 0, U'' < 0$. Lifetime utility[6] is:

$$\begin{aligned} V &= U(C_1) + s(\xi) \cdot \Big\{\{1 - q(\xi)\} \cdot U(C_2^h) + q(\xi) \cdot U(C_2^l)\Big\} \\ &= U(C_1) + \{s(\xi) - l(\xi)\} \cdot U(C_2^h) + l(\xi) \cdot U(C_2^l), \end{aligned} \tag{1}$$

where C_2^h is consumption when healthy and C_2^l is consumption when in need of long-term care at date $t = 2$, and $l(\xi) = s(\xi) \cdot q(\xi)$ is the unconditional probability of becoming in need of long-term care. In line with our later empirical results, we assume that individual types that live longer spend shorter time in long-term care, so that $Corr(s(\xi), l(\xi)) < 0$.

3.1 Stand-alone Annuity and LTC insurance

We first study two stand-alone contracts $k = \{A, L\}$ of an annuity A and a LTC insurance L. Individuals have initial wealth W_t in both periods $t = 1, 2$ where $W_1 > W_2$. In period 1 the agent can buy annuity insurance at premium P_A paying a benefit Υ in $t = 2$ in case of survival, and LTC insurance at premium P_L that covers long-term care costs X in the event of poor health at old age. Hence, the benefit is $B = \{\Upsilon, X\}$ under each insurance. There are no savings in the model so the budget constraint in period 1 is given by $C_1 = W_1 - P_A - P_L$. In period 2, the agent can consume $W_2 + \Upsilon$ in both states if insured. If uninsured, the agent has $C_2^h = W_2$ if surviving healthy and $C_2^l = W_2 - X$ if surviving in need of long-term care.

Rational individuals make a binary choice to buy insurance or stay uninsured, taking

[6] Note, we assume homogeneous preferences implying that the only heterogeneity between households are the two risks.

the other insurance as given.[7] Comparing the expected utility from being insured with the value from staying uninsured, we can derive the willingness to pay $\pi(\xi, k)$ (WTP) for an insurance for each type. With this, define aggregate demand $D_k(P_k)$ for insurance k as the mass of types whose willingness to pay exceeds the uniform price P_k for the insurance product:[8]

$$D_k(P_k) = \int_{\Xi} \mathbb{1}(\pi(\xi, k) \geq P_k) dG(\xi). \tag{2}$$

Risk-neutral insurers have to cover only the costs $c(\xi, k)$ for each insured individual and compete in a Bertrand game over the price of the product. Firms cannot observe individual risk and have to price insurance based on an average risk type and cost AC_k.[9]

The distinguishing feature of the adverse selection model relative to the standard supply and demand model is that supply is not determined with an independent production technology. Instead, the average cost curve –the supply curve– AC_k is given by

$$AC_k(P_k) = \frac{1}{D_k(P_k)} \int_{\Xi} c(\xi, k) \cdot \mathbb{1}(\pi(\xi, k) \geq P_k) dG(\xi) = \mathbb{E}\left\{c(\xi, k) | \pi(\xi, k) \geq P_k\right\}, \tag{3}$$

which is determined by the types who choose to buy insurance.

The marginal cost curve in the market is given by $MC_k(P_k) = \mathbb{E}\left\{c(\xi, k) | \pi(\xi, k) = P_k\right\}$, and it is downward sloping so that marginal costs increase in price and decrease in quantity. This shape is generated by the fact that individuals with the highest willingness to pay for insurance are also those with the highest expected costs, but the type ξ is private information. Further, due to agents being risk-averse, the marginal cost curve locates below the demand curve.

Zero profit implies that the equilibrium insurance premium equals the average costs of the entire risk pool willing to buy the insurance at the given premium, so the firms' information problem implies welfare losses relative to a world with complete information.

Panel (a) in Figure 1 provides a stylized graphical representation of the welfare losses

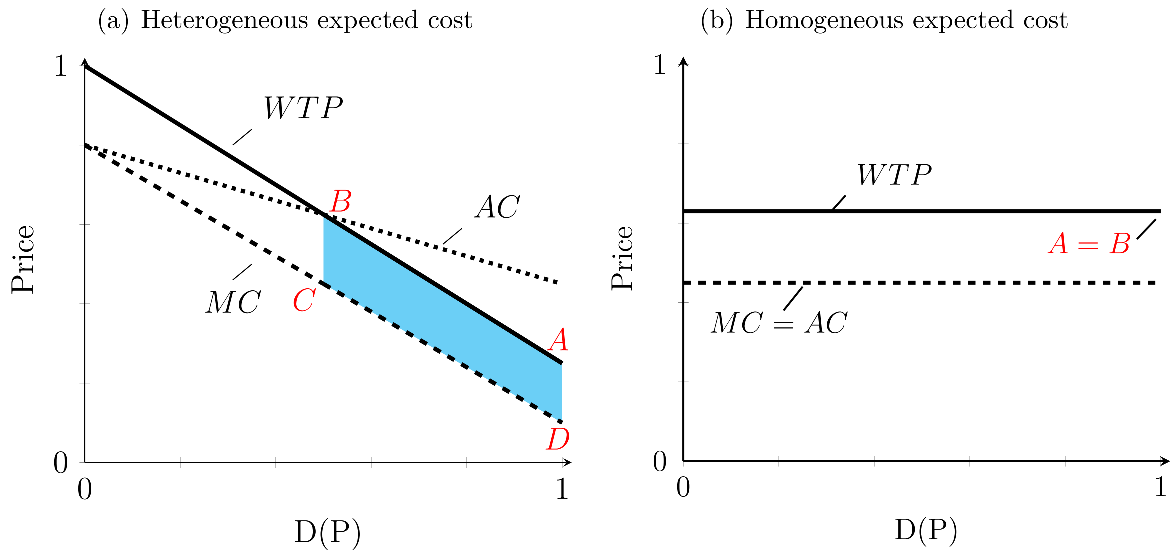
[7] When studying one insurance, we assume that the respective other risk is fully insured so that we only have two groups: insured and uninsured agents. Solomon (2022) provides an extension where agents can decide to buy either insurance, both insurances or to stay uninsured. The main results are not affected by our simplifying assumption.

[8] See Appendix C for the derivation of the demand- and WTP-curve for our two period model and accompanying comparative statics on the slope of the demand curve.

[9] Besides this adverse selection, we abstract from any other friction like, e.g., moral hazard. Firms are also not allowed to compete on the coverage features as in Rothschild and Stiglitz (1976) type models. Webb (2009) explicitly shows in this setup that bundling an annuity and a LTC insurance with negatively correlated risks for these states is a Pareto improvement.

in a market for the stand-alone insurances.[10] In our example, the WTP curve is always above the MC curve due to the assumed risk-aversion, implying that agents always prefer being insured when pricing is at marginal costs. Due to asymmetric information, pricing occurs at average costs. Hence, the equilibrium price is in point B, where the willingness to pay of a new –lower-cost– individual no longer exceeds the average cost of the existing insurance pool. It is optimal for the marginal consumer to remain uninsured. The welfare loss due to asymmetric information is the deadweight loss $\overline{ABCD}$, which equals the sum of risk premia of uninsured individuals who are willing to pay a positive risk premium.

Figure 1: Adverse selection effects with different cost patterns



The slope of the WTP curve is determined by the dispersion of types in the economy: a high willingness to pay for insurance implies a high underlying risk and vice versa. What happens if the heterogeneity in risk decreases? A lower dispersion in costs, $Var(c(\xi, k))$, flattens the WTP-, and the two cost curves. The willingness to pay across agents, as well as their costs, become more aligned. In effect, more agents would be insured (point B would move to the right), and the dead weight loss would decrease. Panel (b) Figure 1 depicts the extreme case without dispersion, $Var(c(\xi, k)) = 0$. With all individuals facing the same expected costs the demand- and supply curves become linear. Average costs are equal to marginal costs but below the WTP-curve due to the risk premium that

[10]We depict linear demand and supply curves which arise if the probabilities are uniformly distributed, which is assumed for the following analysis. Non-linearities, in contrast, arise from normally distributed probabilities.

agents are willing to pay. In that case, the first-best optimum of full insurance in point $A = B$ is possible for every risk-averse individual because asymmetric information no longer plays a role.

Of course, assuming risk-averse agents implies that agents would buy insurance even with a negative return on the insurance due to a positive risk premium that they are willing to pay. This means that the first best allocation is already achieved in this model before adverse selection is completely eliminated. In fact, reducing $Var(c(\xi, k))$ to the point where the WTP curve is above the AC-curve for all agents in Panel (a) of Figure 1 is enough to ensure full insurance. When studying an optimal comination of insurances, we will use the objective to minimize the variance in costs to get results that are independent of household preferences to simplify the analysis.

3.2 Optimal Combined Insurance

The Life Care Annuity In a combined insurance product, the life care annuity CA, agents can pay the premium P_{CA} that pays out the annuity Υ if the agent survives with probability $s(\xi) - l(\xi)$ and is healthy, and the payout is $(1 + \rho)\Upsilon$ if the agent survives but needs long-term care with probability $l(\xi)$.[11] The general idea is to hedge the two risks when $Corr(s(\xi), l(\xi)) < 0$ to attract a higher number of people choosing this insurance. Assume an individual with a high risk for annuities (high life expectancy), implying high costs, and simultaneously with a low risk of long-term care, implying low cost for LTC insurance. A second agent has low life expectancy and high long-term care risk, implying the reversed costs for the two insurances. The variation in the cost, $Var(c(\xi, k)) \neq 0$, implies adverse selection problems for stand-alone products. However, combining the two insurances hedges the risks and aligns the costs of these two agents. In the optimal outcome, the costs of the agents are equal, i.e., $c(\xi, k) = \bar{c}$ and $Var(c(\xi, k)) = 0$, which eliminates the adverse selection problem and makes the first best allocation feasible in our simple model so that everyone is insured, cf. Figure 1(b).

Optimal Combination of Insurance We aim to determine a contract of a combined insurance that maximizes the fraction of insured agents by minimizing adverse selection.

[11] We assume that stand-alone insurance is unavailable when studying combined insurance. This could be labeled as the 'managed competition' case – cf. Solomon (2022) – where a regulator, or market designer does not allow single insurance contracts.

This can be achieved by the appropriate choice of ρ, which governs the relative size of the benefit in each state. Note that the benefit in case of LTC is then no longer restricted to be capped by the long-term care costs X, but we rather allow for arbitrary top-up values ρ, which might exceed the costs. The optimal size of the top-up ρ that minimizes adverse selection in this model is reached if expected individuals' cost are homogeneous for all types ξ:

$$\mathbb{E}\left(c(\xi, \mathrm{CA}, \rho)\right) - \bar{c} = 0. \tag{4}$$

Consider a simple example with only two types $\xi = (1, 2)$. To equalize benefits, the benefit level ρ needs to be chosen such that condition (4) is met for both types, implying that their costs are equal:[12]

$$c(1, CA, \rho) = c(2, CA, \rho) \implies s(1) \cdot \Upsilon + l(1) \cdot \rho \cdot \Upsilon = s(2) \cdot \Upsilon + l(2) \cdot \rho \cdot \Upsilon,$$

Solving for ρ gives:

$$\rho = \frac{s(2) - s(1)}{l(1) - l(2)}.$$

Obviously, $\rho > 0$ if $s(2) > s(1)$ and $l(1) > l(2)$, or vice versa: the time in long-term care $l(\xi)$ and remaining life expectancy $s(\xi)$ have to be negatively correlated to sustain a positive LTC insurance benefit.

If there are infinitely many types, there is no closed-solution possible, and we have to bring the average cost $\bar{c}$ as close as possible to individual expected cost. We do this by minimizing the squared difference of (4):[13,14]

$$\min_{\rho} \mathcal{F}(\rho) = \mathbb{E}\left\{\frac{s(\xi) + l(\xi) \cdot \rho}{\mathbb{E}(s(\xi)) + \mathbb{E}(l(\xi)) \cdot \rho} - 1\right\}^2 = \mathbb{E}\left\{PR\left(\xi, \rho\right)^2\right\} = Var\left\{PR(\xi, \rho)\right\}$$

with

$$PR(\xi, \rho) = \frac{s(\xi) + l(\xi) \cdot \rho}{\mathbb{E}(s(\xi)) + \mathbb{E}(l(\xi)) \cdot \rho} - 1 \tag{5}$$

In the multi-period framework, $s(\xi)$ represents the remaining life expectancy and $l(\xi)$, the unconditional remaining lifetime spent with long-term care needs. PR in Equation (5)

[12]See Appendix C for comparative static on how the slope of the demand curve changes if the correlation between $s(\xi)$ and $l(\xi)$ becomes more negative.

[13]Without affecting our main results, we divided the objective function by $\bar{c}$ so that we can express it in terms of premium returns to get a better intuition for the results.

[14]We here assume a real interest rate of zero so that the time value of money does not play a role in the model.

is the *premium return* of the life care annuity, defined as the difference between the ratio of expected (present) value of benefits relative to its premium. An analogous concept of *money's worth* was suggested by Mitchell et al. (1999) and used by, e.g. Finkelstein and Poterba (2004) and Brown and Finkelstein (2007).[15] In the model discussed above, the expected value of benefits for each type is $s(\xi) + l(\xi) \cdot \rho$. The uniform premium in a competitive market is given by $P_{CA} = \mathbb{E}(s(\xi)) + \mathbb{E}(l(\xi)) \cdot \rho$ which is the denominator of Equation (5). A value of unity implies a premium return of zero: the pricing of the insurance is then actuarial fair with premia equal to the expected value of benefits. Our objective function aims to minimize the variance in premium returns, implying as little heterogeneity in marginal cost as possible. It is important to note that our objective is to minimize welfare loss due to adverse selection. We leave the explicit modeling of the choice of different insurance contracts for future research.

Deriving the first-order condition from the optimization problem (5) and solving for the optimal top-up ρ yields our main result[16]

$$\rho^\star \;= \frac{\mathbb{E}(s(\xi))}{\mathbb{E}(l(\xi))} \cdot \frac{\left\{\frac{\text{SD}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\}}{\text{SD}\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}}\right\} - \text{Corr}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}, \frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}}{\frac{\text{SD}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\}}{\text{SD}\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}}^{-1} - \text{Corr}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}, \frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}}, \tag{6}$$

where $\frac{s(\xi)}{\mathbb{E}(s(\xi))}$ and $\frac{l(\xi)}{\mathbb{E}(l(\xi))}$ can be interpreted as the money's worth of the stand-alone insurances, i.e. one plus the premium return, for the stand-alone annuity-, and LTC insurance, respectively. The optimal size of benefit in long-term care relative to not in long-term care, $\rho^\star$, depends on three main elements: (1) the relative duration in each state, $\frac{\mathbb{E}(s(\xi))}{\mathbb{E}(l(\xi))}$, (2) the relative standard deviations of the money's worth for stand-alone LTC- and annuity insurance, $\frac{\text{SD}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\}}{\text{SD}\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}}$, and (3) the correlation of the money's worth of the two stand-alone insurances, $\text{Corr}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}, \frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}$.

The relative duration in each state, i.e., the expected life expectancy relative to the expected duration in need of long-term care, has a proportional impact on ρ^*. Assuming the relative standard deviations to be one and a perfectly negative correlation, the intuition is straightforward. Let life expectancy be two times higher than the time spend

[15] The money's worth used by Finkelstein and Poterba (2004) is defined as the expected present discounted value of annuity payouts divided by the initial premium. In our terminology, this could be defined as the premium return plus one, equal to one if the benefits align with the premium.

[16] Appendix D provides the derivation.

in need of long-term care so that $\frac{\mathbb{E}(s(\xi))}{\mathbb{E}(l(\xi))} = 2$, then the level effect of optimal condition (6) implies that the top-up of LTC benefits must also be twice as high compared to the state when not in need of long-term care ($\rho^* = 2$) to eliminate the differences in premium returns.

The second factor is a measure for the heterogeneity in risks and can be interpreted as the heterogeneity in the money's worth in each stand-alone insurance. In effect, this measure is an indication which of the two insurances suffer from more severe adverse selection problems. In Section 3 we showed that decreasing heterogeneity in types also decreases the deadweight loss. A value above one implies that the heterogeneity in premium returns is larger for an annuity whereas this is reversed if this ratio is smaller one. The impact of this factor on the optimal combination of the two insurances is again straightforward. Assuming a relative duration of 1 and again a perfectly negative correlation, we also have a proportional effect on ρ^*. If the heterogeneity in premium returns is twice as large when in need of long-term care so that $\frac{\mathrm{SD}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\}}{\mathrm{SD}\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}} = 0.5$, then the implied top-up is given by $\rho^* = 0.5$. The intuition is that the optimal combination of the two insurances implies that a higher benefit should be granted in the state where heterogeneity in risks is lower. Finally, note, that taken both factors together assuming a perfectly negative correlation simply consist of the product of the two:

$$\rho^*_{\mathbf{Corr}=-1} = \frac{\mathbb{E}(s(\xi))}{\mathbb{E}(l(\xi))} \cdot \frac{\mathrm{SD}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\}}{\mathrm{SD}\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}}. \tag{7}$$

This equation shows that the two factors can offset each other: following the example above, if there is more heterogeneity in long-term care risk but the duration is shorter than surviving healthy, then it might be optimal to combine an annuity and a LTC insurance and pay out the same benefit in both states, i.e., $\rho^* = 2 \cdot 0.5 = 1.0$.

The third factor measures the correlation between the money's worth of the two stand-alone insurances. When the risks – and hence the premium returns of the stand-alone insurances – are not perfectly negatively correlated, the combination of the two insurances can only partially eliminate adverse selection incentives. The risks cannot be perfectly hedged and there are remaining differences in the premium returns in a combined insurance. Besides, a correlation between $[-1, 0]$ reinforces Equation (6)'s first two effects on the optimal top-up ρ^* in both directions. Hence, a lower correlation in absolute terms yields a positive (negative) effect on ρ^* if the ratio of the standard deviation (factor 2 in Equation (6)) is larger (less) than one.

Bringing the Model to the Data In our empirical section, we will elaborate on the quantitative importance of the effects described above for the different groups ξ, which we will specify as quintiles of lifetime income, gender, and marital status. Section 5 shows results for premium returns over lifetime income quintiles. We will label the slope of the line connecting the premium returns over income quintiles as *gradients*, referring to the well-known socioeconomic gradient in mortality discussed in the health-economics literature, see, e.g., Dow and Rehkopf (2010). Factor two in Equation (6) – the ratio of the standard deviations – is a measure for the sign and the steepness of these gradients. In contrast, the correlation can be seen from the shape of the premium returns over income and their relative (opposing) slopes: two linear and opposing slopes indicate a high negative correlation.

We further assume that agents can purchase insurance by paying a lump-sum premium P_k at (initial) age 65, priced at the average risk. We estimate the quantities $s(\xi)$ and $l(\xi)$ with our multi-state model. We discretize type distribution G, by taking the empirical probability of observing the type ξ at age 65. This also allows us to calculate the population's remaining life expectancy $\mathbb{E}(s(\xi))$ and the unconditional time spend in long-term care $\mathbb{E}(l(\xi))$.

4 Data and empirical approach

4.1 Institutional context

The Netherlands has a universal and generous pension and long-term care system. The pension system consists of a tax-funded minimum social security benefit (first-pilar) that is paid from the statutory retirement age to every Dutch citizen with a required minimum time living in the country. This AOW (*Algemene Ouderdomswet*) pension is complemented with a (second-pilar) occupational defined benefit pension, which is mandatory (except for self-employed) and based on past lifetime earnings. The replacement rate is quite high, reaching around 70% of average lifetime earnings (Knoef et al. (2017)).

The public long-term care system provides coverage for both formal long-term care at home and in a nursing home. Unlike the U.S., private LTC insurance and out-of-pocket expenditures are marginal, being less than 0.5% of total long-term care expenditures (Colombo et al., 2011). Everyone who lives in the Netherlands is insured and pays income-dependent premia. Total long-term care expenditures are 4.1% of GDP and among the highest of OECD countries (European Commision, 2015). Every request for long-term care is assessed by the Centre for Care Assessment (CIZ), taking into account the usual informal care that partners or other household members give to each other (Mot, 2010). Nursing home care is available for individuals with more severe conditions or a less supporting environment. However, individuals may also choose to receive personal care at home. When getting personal care at home, the partner is expected to provide the usual domestic and supportive care. Individuals are entitled to less personal care when the partner voluntarily provides personal care (Mot, 2010; Bakx et al., 2015). In 2015, a major long-term care reform has been implemented, reducing coverage and increasing co-payments. In the new system, only people who need care day and night are entitled to care in a nursing home. For people with lighter care needs, personal care at home is no longer publicly insured (Maarse and Jeurissen, 2016).

Overall, the Netherlands stands out from other OECD countries in old-age social insurance by providing an almost universal public long-term care scheme with generous coverage, which implies low out-of-pocket expenses so that adverse selection problems for using long-term care are arguably low. Eligibility rules depending on informal care availability also suggest low selection effects into long-term care. These institutional factors allow us to estimate arguably unbiased socioeconomic differences in long-term care use and mortality.

4.2 Data and sample selection

We use administrative data for the Netherlands containing detailed longitudinal information on formal long-term care use and mortality (exact date of death) for the entire population. Administrative data on formal nursing home care and home care is obtained from the Central Administration Office (CAK). These data cover all residents of the Netherlands aged 18 and older who have long-term care expenses that are covered by the public long-term system. Data on mortality is obtained from the causes of death registry. In addition, we use detailed income and assets data from tax registries to measure socioeconomic status. Demographic characteristics, including age, gender, and marital status are obtained from the municipality population register.

While the long-term care use data are available since 2004, we use them starting in 2006 when also assets data are available to determine socioeconomic status. Our study ends in 2014 before the major reforms of the long-term care system were implemented. We include retired individuals aged 65+ and their partners whose main source of income is pension income. We exclude individuals if they are not registered in the Netherlands for the entire sample period. Further, we exclude households remarrying or divorcing after age 65 (4.5%). We exclude a few households with negative income or assets and those with missing data (0.2%). This leaves us with a final sample of 3,219,297 individuals in 2,198,755 households.

4.3 Variables

Formal long-term care use is defined broadly, including institutionalized and home care. Institutionalized care comprises nursing home care and psychiatric or disabled care. For our sample, nursing home care covered about 93% of institutionalized care in 2006. Home care use is defined as receiving personal care, such as help with daily activities (ADL), and nursing care, such as wound dressing. We do not include domestic care. For institutionalized care, we measure each spell's starting and end date; for home care we measure the spells on a 4-week basis after 2008 and until 2008 as the first and last day of use in the year.[17] We excluded spells where home care was provided for less than one hour during the year.

For the covariates, marital status is defined as being in a couple (married, a registered partnership, or cohabiting) or a single-person household. Socioeconomic status is measured by average retirement income, which is the sum of personal gross income (deflated using CPI) – and for couples, its sum – and the annuity value of household financial assets. As our sample contains retired individuals only, average retirement income provides a good proxy for lifetime income. To compute the annuity value of household assets, we follow Knoef et al. (2016), see Appendix A for details. Household financial assets are particularly important to include as a source of retirement income for former self-employed individuals. Retirement income is equivalized using OECD scales to make

[17]For 2008, each spell's start and end date is marked by the start and end of the calendar year.

couples and single-person households comparable regarding retirement income. Based on this measure, we construct lifetime income quintiles.

4.4 Multi-State Model

We use a multi-state model to estimate lifetime long-term care use and remaining life expectancy at age 65 for different groups $h \in \mathcal{H}$ (lifetime income quintile, gender, initial marital status at age 65). The model has three states (no long-term care use, long-term care use, and death) with transition rates $\lambda_k(t)$, and individuals can repeatedly visit the states (both in the model and data). To estimate the transition rates we apply a competing risk analysis, i.e. we take into account that only one of two possible transitions takes place, leaving the other transition unobserved. We assume the transition rates to be independent in terms of unobservable characteristics, so the transition rates can be separately estimated per state using a mixed proportional hazard (MPH) model (Hougaard, 2000; van den Berg, 2001):

$$\lambda_k(t, marstat_i(t); \nu_i^k, \boldsymbol{\gamma}_k, \boldsymbol{\beta}_k) = \lambda_0(\boldsymbol{\gamma}_k, t) \cdot \phi(\boldsymbol{\beta}_k, marstat_i(t)) \cdot \nu_i^k, \tag{8}$$

where $\lambda_0(\boldsymbol{\gamma}_k, t) = \exp\{(\gamma_k + \gamma_{kh}) \cdot t\}$ is the baseline hazard capturing age-specific transition rates for each state and group, with t as the age-indicator. The parameter γ_{kh} captures the difference in the age-specific transition rates over groups. The advantage of using age as a time scale is that we abstract from unknown information regarding some individuals' beginning of the no-long-term care use or long-term care use spell. Otherwise, we should have imputed the starting dates or excluded these left-censored spells[18], which might result in biased estimates because of an initial conditions problem Heckman (1981). We assume a Gompertz functional form for the baseline hazard, which is a common specification for adult mortality in developed countries (see e.g. Missov et al., 2015).

The second term of the MPH model $\phi(\boldsymbol{\beta}_k, marstat_i(t)) = \exp\{\ \beta_k + \beta_{1kh} + \beta_{2kh} marstat_i(t)\}$ includes current marital status (for initially married couples) as a time-varying covariate to capture the transition from being married to a single-person household. Moreover, it captures differential mortality and differences in informal care possibilities between singles and couples. The parameter β_{1kh} measures the difference between initial singles and

[18]Contrary to left-truncated spells, left-censored spells have an unknown start date.

initially married individuals who have become single, and β_{2kh} picks up the additional impact of becoming single while currently married.

The third term of the MPH model $\nu_i^k \sim \Gamma\left(\frac{1}{\sigma_k^2}, \frac{1}{\sigma_k^2}\right)$ is an individual-specific random effect accounting for dynamic selection and other unobservable differences between individuals, for instance, factors explaining mortality among the oldest old and the mortality plateau (see e.g. Vaupel et al., 1998; Barbi et al., 2018). We assume this so-called frailty term to follow a Gamma distribution because it well describes observed heterogeneity over long durations (and, therefore, frailty in old age) (Abbring and van den Berg, 2007); moreover, unique parameter identification exists (Honoré, 1993). Individuals draw the random effect value at initial age 65. For tractability, the random effect is not shared over different states.

Estimating a mixed proportional hazard model with left truncation and frailty is computationally challenging because the left-truncated sample has a different frailty distribution. Allowing for time-varying covariates and repeated spells adds a layer of complexity. Because we assume independence across transitions, we follow the estimation technique by van der Vaart and van den Berg (2023) addressing these challenges; see Appendix B for more details and the maximum likelihood specification. Having estimates on the transition rates, we use a simulation model to determine long-term care use and remaining life expectancy at age 65 for different groups. As a starting point, we use the conditional distribution of our variables at age 65 (see Table 4 in Appendix B). For the simulations, we extend the approach by Crowther and Lambert (2017) to allow for transitions from couples to single-person households. More specifically, for couples, we first simulate age profiles from age 65 until the end of life for both partners. Next, we re-simulate the remaining age profile for the surviving partner according to our simulation model. We simulate N=100,000 households repeated 5,000 times to construct 95% confidence intervals; see Appendix B for additional details.

5 Results

We show results on the simulated durations of long-term care and life expectancy over lifetime income and we highlight the importance of gender and marital status. We then show how these differences translate into the value of annuity- and LTC insurance and we finally present results for a life care annuity.[19]

5.1 Socioeconomic Differences in Long-term Care and Mortality

We find substantial gradients in long-term care use and remaining life expectancy over lifetime income. Table 1 shows that low-income individuals live shorter than high-income individuals but use more long-term care. On average, men and women in the bottom income quintile, respectively, live 4.0 and 2.3 years shorter than their high-income counterparts in the top income quintile (see last column). On contrary, low-income men and women spend 1.1 and 1.7 years longer in long-term care than their high-income counterparts. There are also gradients in the probability of ever using long-term care, ranging from 91% for women in the bottom income quintile to 86% in the top income quintile. Overall, the income gradient concerning life expectancy is steeper for men than for women while, reversely, the gradient for long-term care is steeper for women than for men.

Table 1: Life Expectancy and Long-term Care Use by Lifetime Income Quintiles

(a) Men	All	Bottom	Second	Third	Fourth	Top	ΔTop - Bottom
LE at age 65 (years)	18.0 (17.9;18.1)	15.3 (15.0;15.5)	16.8 (16.6;17.0)	17.6 (17.5;17.8)	18.4 (18.2;18.6)	19.2 (19.1;19.4)	4.0 (3.7;4.2)
Δ(Married - Singles)	2.5 (2.3;2.7)	2.0 (1.6;2.5)	2.7 (2.2;3.1)	2.6 (2.2;3.0)	2.5 (2.1;2.9)	1.6 (1.2;2.0)	−0.4 (−1.0;0.1)
LTC (years)⋆	3.1 (3.0;3.1)	3.8 (3.7;4.0)	3.4 (3.3;3.6)	3.2 (3.1;3.2)	3.0 (2.9;3.0)	2.8 (2.7;2.8)	−1.1 (−1.2;−0.9)
Δ(Married - Singles)	−0.8 (−0.9;−0.7)	−1.8 (−2.0;−1.5)	−1.5 (−1.8;−1.3)	−0.7 (−0.9;−0.6)	−0.3 (−0.5;−0.2)	−0.2 (−0.4;−0.1)	1.6 (1.3;1.9)
Ever use LTC (%)	77 (76;78)	79 (78;80)	79 (78;80)	78 (77;79)	77 (76;78)	75 (75;76)	−3 (−5;−2)
(b) Women							
LE at age 65 (years)	21.9 (21.8;22.0)	20.1 (19.9;20.3)	21.8 (21.6;22.0)	22.0 (21.8;22.2)	22.2 (22.1;22.4)	22.3 (22.2;22.5)	2.3 (2.0;2.5)
Δ(Married - Singles)	1.8 (1.6;2.0)	3.0 (2.6;3.4)	2.0 (1.6;2.4)	1.4 (1.0;1.7)	1.2 (0.8;1.5)	0.6 (0.3;1.0)	−2.4 (−2.9;−1.9)
LTC (years)⋆	5.1 (5.1;5.2)	6.0 (5.9;6.2)	5.9 (5.8;6.0)	5.3 (5.2;5.4)	4.8 (4.7;4.9)	4.4 (4.3;4.4)	−1.7 (−1.8;−1.5)
Δ(Married - Singles)	0.1 (0.0;0.1)	0.4 (0.2;0.7)	0.4 (0.2;0.6)	0.3 (0.1;0.5)	0.4 (0.2;0.6)	0.4 (0.2;0.6)	−0.1 (−0.4;0.3)
Ever use LTC (%)	89 (88;89)	91 (91;92)	91 (91;92)	89 (89;90)	88 (88;89)	86 (85;87)	−5 (−6;−4)

Notes: These are population-averaged measures for the life cycle simulation of 100,000 households. We present the median estimates across 5,000 bootstrapped samples and the 2.5th and 97.5th percentiles between brackets. Sample sizes are reported in Table 4 in Appendix B.

To see the role of having a partner for these socioeconomic gradients, we turn to the difference for initially married versus initially single individuals. Marital status is an

[19] A robustness check confirmed a good match between simulated and empirical surival and long-term care use probabilities by age, marital status, lifetime income and gender. Results are available upon request.

important factor influencing the transition into long-term care and mortality. We simulate the durations separately for individuals who married at age 65 and those single and compute the difference $\Delta(Married - Singles)$.The difference in life expectancy between initial married and singles is 2.5 years for men and 1.8 years for women. This survival advantage of being married is among others reported in Pijoan-Mas and Ríos-Rull (2014). In addition, we find that single men spend 0.8 years more in long-term care than their married counterparts. We do not find a significant difference of long-term care use over marital status for women. This result suggests that women have fewer opportunities to get informal care from their spouse than men and tend to live longer.

The number in the last column corresponds to a differences-in-differences approach showing how being married or single affects the difference between the top and the bottom income quintile. Our results show that the gap in life expectancy between the bottom and top income group is 2.4 years smaller for married women than for single women. Essentially, this implies that being married flattens the income gradient of life expectancy for women: only for single women we observe a strong gradient over income while this is moderate for married women. This same number is only 0.4 years for men, implying that the gradient in life expectancy is only moderately flattened for married individuals. Similarly, the gap in long-term care use between the bottom and top income group is 1.6 years smaller for married men than for single men. This number is 0.1 years but insignificant for women. Again, this implies that the income gradient in long-term care use for married men is almost flat, whereas it is relatively strong for single men.

Turning to the socio-demographic difference, we find that women tend to live 3.9 years longer than men (21.9-18.0 years). In addition, women have a higher prevalence and longer duration of long-term care use than men: About 89 percent of women ever uses long-term care with an average duration of 5.1 years conditional upon use. In contrast, 77 percent of men use long-term care with an average duration of 3.1 years, amounting to 12% of their remaining lifetime, compared to 18% for women.

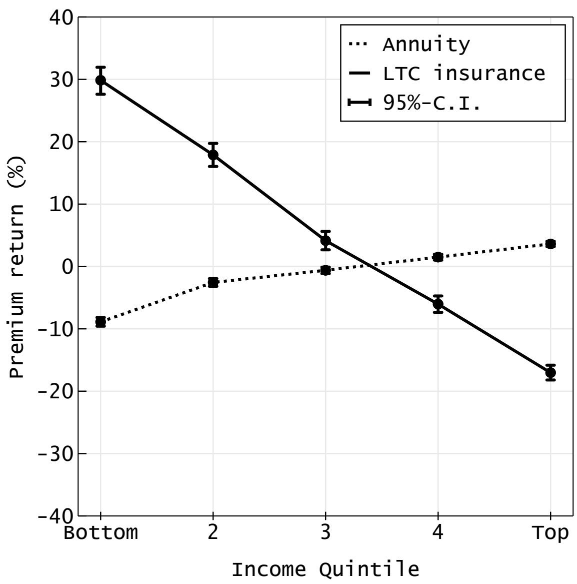
5.2 Premium Returns of Old-Age Insurances

5.2.1 Stand-Alone Contracts of Annuity and LTC Insurance

We translate the heterogeneity in long-term care use and life expectancy into a money's worth for the different insurances over subgroups according to Equation (5).

Uniform Premium We first study an annuity and a LTC insurance independently assuming a uniform premium for everyone for each insurance, implying that the total insured sample $\mathcal{H}$ comprises the whole population at age 65+. We focus on the income quintiles as our subgroups. The implied premium returns are depicted in Figure 2.

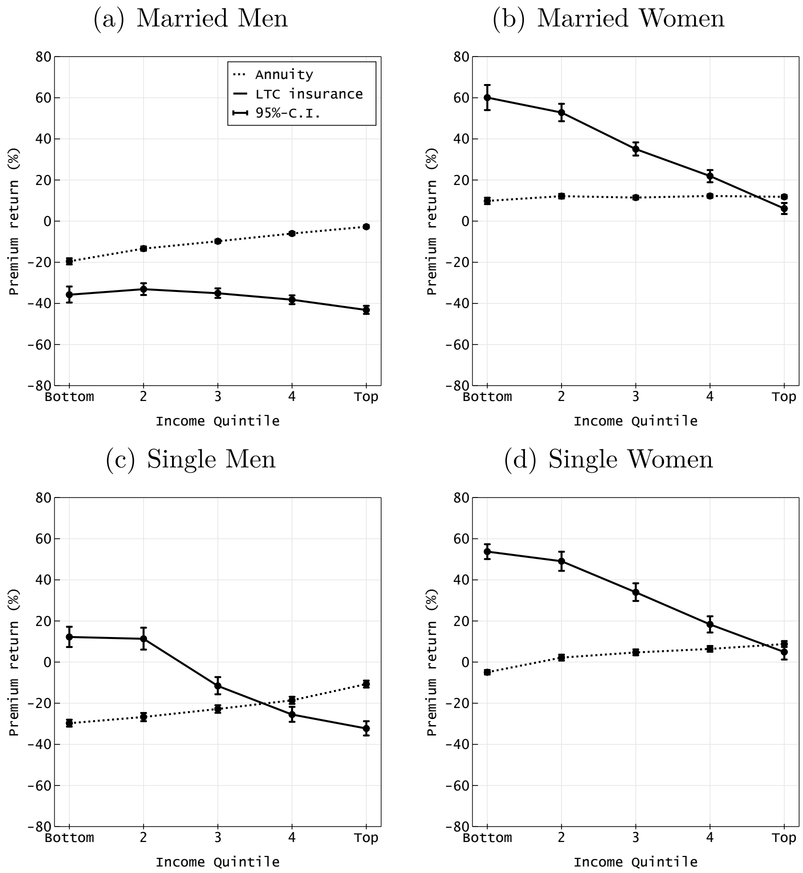
Figure 2: Premium Return with Uniform Premium



Notes: Population-averaged premium returns for the life cycle simulation of $100,000$ individuals. Medians across 5,000 bootstrapped samples are shown. The underlying premium returns on the pension annuity, LTC insurance, and life care annuity are provided in Table 8 in Appendix F.

As reflected by the steeper line, the results show that benefit inequality across income groups is larger for LTC insurance than for annuities. The premium return for the lowest income group is 29.9 percent, implying that a premium of one Euro yields an expected value of benefits of 1.299 Euro. On the other hand, the highest income groups lose 17.0 cents on every euro invested in the LTC insurance. On the contrary, for every euro invested in the annuity priced according to the average risk, households in the lowest income group receive only 91.1 cents. Households in the highest income groups have a positive return and earn 3.6 cents on top of every euro invested. The larger discrepancy in premium returns for LTC insurance makes this insurance product more prone to adverse selection by income groups than pension annuities in our case.

Figure 3: Premium Returns by Gender and Marital Status with Uniform Premium



Notes: These are population-averaged premium returns for the life cycle simulation of 100,000 individuals. Medians across 5,000 bootstrapped samples are shown.

The socio-demographic differences of life expectancy and long-term care use over marital status and gender translate into heterogeneity in premium returns over these dimensions. There is also a negative correlation in risks for marital status as married individuals live longer but spend less time in long-term care - at least for men. Note, however, that the two risks are not negatively correlated over gender because women have a higher life expectancy and spend more time in long-term care.

This is reflected in Figure 3 which shows the implied premium returns with uniform premium over marital status and gender. The large difference across panel (a)-(d) reveals strong level effects, particularly over gender. Married men have negative premium returns throughout the income distribution, whereas married women value both insurances. The reason for this outcome is simple: men die earlier and they use less long-term care. Insurances priced at the average risk are not valuable for this group.[20] The picture is similar for singles, except single men in the two lower income quintiles who enjoy positive returns of an LTC insurance.

Group-Specific Premia The large differences in premium returns for stand-alone pension annuities and LTC insurance can potentially lead to strong adverse selection effects based on marital status and gender. To prevent a potential unraveling of the insurance market, group-specific premia based on observables such as marital status and gender might reduce adverse selection problems.

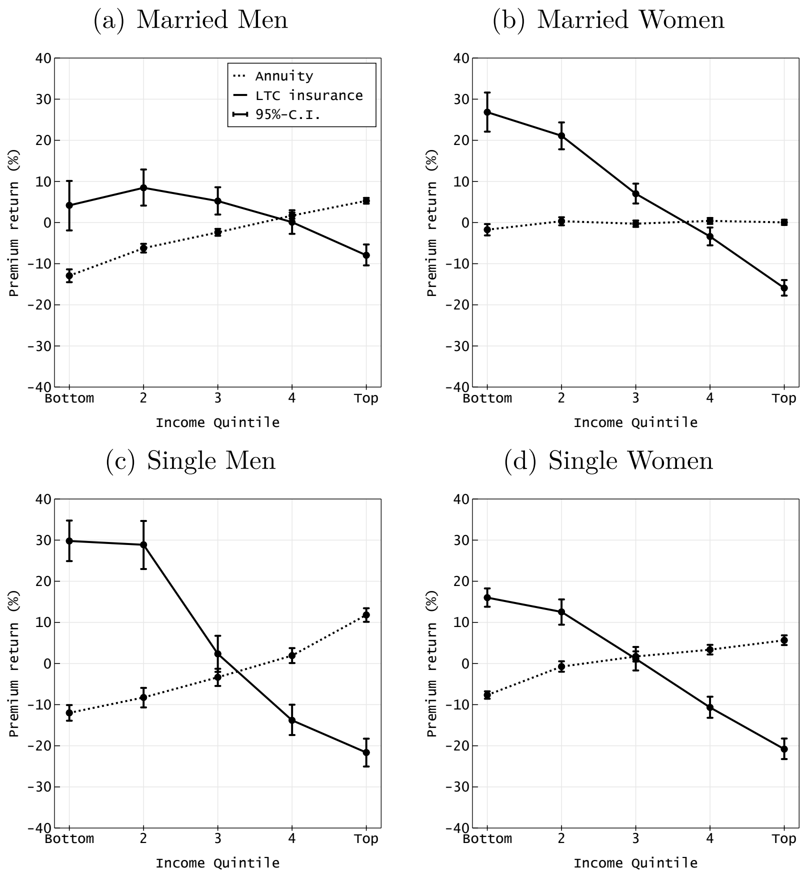
Figure 4 shows the effect of marital-status-, and gender-specific premia on the premium returns. Compared to Figure 3, group-specific premia shift the lines closer to zero, while –unsurprisingly– the gradients over income still persist. Offering premia that may differ over gender and marital status, however, are able to eliminate the large level effects of the premium returns between these groups which decrease the adverse selection problem significantly. Figure 4 also shows large variations in the steepness and the shape of the gradients. For example, the income gradients in long-term care are particularly steep for married women and single men, while the shape of the gradient for married men is more hump-shaped. The annuity gradient over income is stronger for men (both married and single) and almost non-existent for married women. These differences become important when analyzing the optimal combination of the two insurances which we turn to next.

5.2.2 A Life Care Annuity

As shown in Section 3, combined insurance can moderate welfare losses from adverse selection when the correlation between surviving and getting in need of long-term care is negative. In our setting, this is reflected by the reverse gradients of the premium-return lines depicted in Figures 2 to 4. However, at least with a uniform premium, we

[20] For practical reasons, we assume that discrimination over lifetime income is not possible for insurance companies. This information is not only hard to obtain for insurances, it is also hard to imagine a regressive premium system where the income-poor need to pay higher premia than the income-rich to reduce differences in premium returns in the LTC insurance.

Figure 4: Premium Return over Gender and Marital Status with Group-Specific Premium



Notes: Population-averaged premium returns for the life cycle simulation of 100, 000 individuals. Medians across 5,000 bootstrapped samples are shown. The underlying premium returns on the pension annuity and LTC insurance are provided in Table 8 in Appendix F.

have a positive correlation of longevity and LTC risk over gender, which counteracts this negative correlation (cf. Figure 3).

We derive an optimally life care annuity according to Equation (6) and compare two cases assuming (i) a uniform premium over all observable groups (i.e., lifetime income, gender, marital status) and (ii) group-specific premia over gender and marital status where the optimal top-up ρ^* is found only over the remaining differences over lifetime income.

Table 2 shows the results of the optimal top-up of long-term care benefits and Table 3 presents standard deviations for stand-alone insurances and the life care annuity as a measure for the adverse selection problem with each of the three insurances.

We first turn to the case assuming a uniform premium paid by all individuals. The optimal top-up of LTC benefits needed to minimize the heterogeneity in premium returns across income groups is negative, $\rho^* = -0.56$, implying a lower benefit when needing long-term care, which is, of course, not a meaningful insurance. The result is coming from an overall strongly positive correlation between the two risks across the studied risk types. Recall our finding that women live longer and use more long-term care than men, implying a positive correlation of risks across gender. These large gender differences in longevity and long-term care use are stronger than the – negatively correlated – differences over income and marital status and induce an overall positive correlation between longevity and long-term care use. In addition, Table 3 shows that the standard deviation for the combined product is still very high, so the bundling does not reduce the adverse selection problem by much. Overall, this implies that a life care annuity with uniform premium does not work, so we now turn to group-specific premia.

First, note that with group-specific premia, all correlations turn negative, cf. column 3 in Table 2, which was already implied by the inverse gradients shown in Figure 4. To understand the different values of the optimal top-up, ρ^*, over these groups, let us decompose it into its components in the first three columns. The first column can be interpreted as the value of ρ^* if the heterogeneity in risk would be equal over states (implying a ratio of the standard deviations of one), and the correlation would be perfectly negative. Similarly, the second column would be the value of ρ^* if the duration would be equal for both states and the correlation -1. The value $\rho^\star_{\text{Corr}=-1}$ then is simply the product of the two, while the final column shows the sum of all three effects including the effect stemming from a non-perfectly negative correlation of the two risks.

Turning first to married men we observe that the duration in long-term care is rather short, so the optimal top-up would be 8.5 from the level effect alone. This can be seen from the optimality condition in Equation (6), prescribing a higher benefit to be paid in states with shorter duration. At the same time, the heterogeneity in longevity risk is larger, reinforcing the effect on the optimal top-up. If the heterogeneity effect is isolated, the optimal top-up would only be 1.26 because Equation (6) prescribes to put a higher weight on the less heterogenous state (needing long-term care in this case). The combined effect in column 4 is actually quite close to the final optimal value of 11.16 because the correlation between the two risks is quite strongly negative (-0.73). The value of $\rho^* = 11.16$ implies that the benefit in the case of long-term care need to be more than 11 times larger than the annuity benefit, a high number that we put into perspective in the next section.

Table 2: Optimal life care Annuity: ρ^* and Components

	Level effect	Heterogeneity in risk	Correlation between risks	Optimal LTC top-up	
	$\frac{\mathbb{E}(s(\xi))}{\mathbb{E}(l(\xi))}$	$\frac{\mathrm{SD}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\}}{\mathrm{SD}\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}}$	$\mathrm{Corr}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}, \frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}$	$\rho^{\star}_{\mathrm{Corr}=-1}$	$\rho^{\star}$
Uniform Premium	5.68	0.33	0.57	1.88	−0.56
Group-Specific Premium					
Married Men	8.50	1.26	−0.73	10.67	11.16
Married Women	5.03	0.04	−0.32	0.22	0.08
Single Men	5.25	0.42	−0.93	2.18	2.11
Single Women	4.41	0.35	−0.90	1.55	1.47

Median estimates across 5,000 bootstrapped samples. Optimal top-up and its components according to eq. (7) and (6).

In stark contrast, married women have an optimal top-up of only 0.08 implying the optimal combination of insurances is close to a mere annuity. Two factors from the data drive this result: First, the heterogeneity in risk is very low for annuities compared to a strong one for LTC insurance implying the ratio to be 0.04; also when compared with the flat gradient for annuities and the strong gradient in long-term care insurance in panel (b) of Figure 4. In addition, the correlation is with −0.32 only moderately negative and a combination of the two insurances is not well-suited.

The picture is quite different for single individuals. Here, we find almost perfectly negative correlation between the risks as well as offsetting level effects and heterogeneity in risks yielding reasonable values for the optimal top-up between 2.11 for single men and 1.47 for single women.

The standard deviations for group-specific premium returns in Table 3 reveal the highest values for the stand-alone LTC insurances implying that adverse selection problems are most severe for this case. With group-specific premia, a life care annuity reduces these heterogeneities substantially always yielding lower standard deviations than in both of the stand-alone insurances.

Our results suggest that a life care annuity to hedge the two risks of longevity and long-term care is not quite possible for married men and women. The implied top-up of the benefit in the long-term care state is unreasonably high for married men and

Table 3: Standard deviations of premium returns

	Annuity	LTC insurance	Life Care annuity
	$\mathrm{SD}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\}$	$\mathrm{SD}\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}$	$\mathrm{SD}\left\{\frac{s(\xi)+\rho^*\cdot l(\xi)}{\mathbb{E}(s(\xi))+\rho^*\cdot\mathbb{E}(l(\xi))}\right\}$
Uniform Premium	11.47 (11.21;11.72)	34.64 (33.82;35.45)	11.00 (10.77;11.23)
Group-Specific Premium			
Married Men	7.57 (6.86;8.31)	6.04 (4.32;7.96)	2.42 (1.02;4.15)
Married Women	0.63 (0.31;0.98)	14.15 (12.88;15.37)	0.55 (0.22;0.93)
Single Men	8.74 (7.79;9.72)	21.06 (18.84;23.29)	2.22 (1.09;3.43)
Single Women	5.01 (4.46;5.55)	14.26 (13.0;15.55)	1.64 (0.97;2.34)

Values computed correspoind to the objective function from eq. 5 and multiplied with 100%. Median estimates across 5,000 bootstrapped samples and the 2.5^{th} and 97.5^{th} percentiles between brackets.

unreasonably low for married women. In contrast, a combined insurance is well-suited for single individuals.

6 Discussion

Our analysis points to a broader question of why, in practice, certain risks are covered under bundled policies while others are not. Examples for bundled insurance policies are not only life care annuities, but also life-insurances with a LTC rider, combined disability coverage, reverse mortgage, or home-car insurance, cf. Eling and Ghavibazoo (2019).

In our analysis, we shed further light on when and how to combine insurance products by disentangling the determinants of the risk structure when bundling is possible and what it depends on. To minimize the adverse selection problem, we show that it is not sufficient to only focus at the correlation between lifetime long-term care use and life expectancy, but rather also take into account the average size and variation of these correlated measures.

Our formula for ρ^* is easy to apply and to compare to other studies that report lifetime long-term care use and remaining life expectancy by socioeconomic group. For example, we can approximate a value of ρ^* using results from Ko (2022), Table 4, which documents longevity and long-term care needs over income deciles. Using these numbers yields a value of $\rho^* = 2.09$ for 60+ individuals in the U.S. ignoring the heterogeneity in gender and marital status.[21]

What can be learned from our analysis for the optimal top-up value ρ^* for the life care annuity market in the U.S.? According to www.annuity.org, the monthly income stream paid when healthy in a typical life care annuity contract can be two to three times as large in the case of long-term care needs. This would imply a value of ρ of two or three for a typical life care annuity. According to our results in Table 2 these values are very close to the optimal top-up for single men and women, implying that for these groups the existing insurances in the U.S. would largely diminish adverse selection problems. However, the picture looks quite different for married men and married women: Men would require a benefit level 11 times higher than the annuity paid when not needing long-term care. This is not offered as a combined product and rather resembles a stand-alone LTC insurance. In contrast, a stand-alone annuity would rather fit for married women, which is implied by the value for ρ^* close to zero. Consequently, the current market for life care annuities does not seem to reduce adverse selection problems for married individuals.

Another important dimension that our study highlights are group-specific premia, in particular discrimination of premia over marital status and gender. In the U.S., discrimination over marital status are common practice by offering so-called 'couple discounts'. Solomon (2022) reports couple discounts for LTC insurance of around 25% compared to singles. Different premia also prevail for life care annuities, life insurance, and private annuities. Gender-based pricing in insurance is still practice for many insurances and many states in the US, although the Affordable Care Act banned discrimination over gender for health insurance in 2014. In the European Union, the Court of Justice declared gender-specific premia invalid with European legislation and prohibited this practice in Europe in 2012. For LTC and combined products, however, premia largely vary over gender and marital status, although couples tend to be insured jointly. According to American Association for Long-Term Care, premia for single women are around 50% higher than for men and per-capita also higher than for the combined premium for couples.[22]

We find sizable differences in the heterogeneity in risks over gender and marital status, which calls for the need to discriminate premia over these dimensions to tackle adverse

[21] In the computation we assumed an equal weight for each income group.

[22] See: `https://www.aaltci.org/long-term-care-insurance/learning-center/ltcfacts-2022.php`

selection problems adequately.

7 Conclusion

We quantify socioeconomic and socio-demographic differences in mortality and long-term care by estimating a flexible multi-state model on rich administrative data from the Netherlands. We use the estimated model to examine the adverse selection problems of stand-alone annuities and of LTC insurance for different groups. We further determine the optimal combination of these two products in a life care annuity that reduces the heterogeneity of premium returns across socioeconomic groups. We find a strong socioeconomic gradient in mortality and long-term care implying a negative correlation between the two risks and a large gender gradient in these two risks inducing a positive correlation. A third important factor influencing these differences is marital status indicating the importance of the availability of informal care by the spouse, particularly for men. A life care annuity aiming to minimize the heterogeneity of benefits between socioeconomic groups is not feasible with a uniform premium. Only with group-specific premia and then mostly for single individuals rather than for the married, a life care annuity can reduce adverse selection problems. Our results might provide an explanation for why the existing market for life care annuities in the U.S. is so small.

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Appendices

A Constructing a measure for lifetime income

To compute a measure for lifetime income for the households, we follow Knoef et al. (2016). Their approach allows us to include annuity value of household's financial assets. Some households have low income but many assets, e.g., former entrepreneurs, making it indispensable to include the annuity income from financial assets in a lifetime income definition. We measure lifetime income as the average income during retirement plus the annuity value of financial assets.

We use the population tax files on income (2003-2014) and assets (2006-2014). Assets comprise the sum of savings and stock and bond holdings, but exclude home ownership because this is strongly correlated with not being in a nursing home (read: long-term care). Income is measured at the household level, including labor and business income, retirement income (social security benefits, employer-based, and private pension arrangements), social insurance benefits, taxes, and social insurance contributions. Income predominantly consists of retirement income, because we restrict households to have this as their main source of income.

Yet, we do not observe the annuity value of assets, B, which we will therefore impute. We assume that a household bought an annuity when the oldest member was 65. If available, the other member might be younger than 65. The price of the annuity equals the household's current assets A.[23] The annuity yearly pays B if it is a single-person household and $\sqrt{2} \cdot B$ if it is a couple household. $\sqrt{2}$ is an equivalence scale (OECD, 2011) that the OECD applies when comparing income between single and couple households. $\sqrt{2}$ reflects economies of scaling: couples need less than twice the benefit for singles to reach the same welfare level.

The product is actuarially fair: the benefit level B is set to equal expected lifetime benefits to the current assets A. Hence the benefit level B is household-specific. Expected lifetime benefits look as follows:

$$\mathbb{E}(Benefit(B)) = \begin{cases} \sum_{n=0}^{99} \frac{1}{(1+r)^n} \left(B \cdot {}_n s_m\right) & \text{if Single man at age 65} \\ \sum_{n=0}^{99} \frac{1}{(1+r)^n} \left(B \cdot {}_n s_w\right) & \text{if Single woman at age 65} \\ \sum_{n=0}^{99} \frac{1}{(1+r)^n} \Big(B \cdot {}_n s_m \cdot (1 - {}_n s_w) + B \cdot {}_n s_w \cdot (1 - {}_n s_m) & \\ \qquad + \sqrt{2} \cdot B \cdot {}_n s_m \cdot {}_n s_w \Big) & \text{if Married couple at age 65,} \end{cases}$$

where n refers to the years since the oldest household member turned 65. ${}_n s_m$ and ${}_n s_w$ are

[23]We implicitly assume that households do not save or dis-save after retirement.

the probabilities that the man or woman in the household survives n years after buying the annuity. $1 - {}_n s_m$ and $1 - {}_n s_w$ are the probabilities that the man or woman died within n years after buying the annuity. The probabilities are gender-, cohort-, and age-specific, and taken from the life tables of Statistics Netherlands.[24] These probabilities are age-specific because couple members might have a different age when buying the annuity.

The expected benefits $\mathbb{E}(Benefit(B))$ are the sum of benefits expected in each period. We assume a maximum benefit payout period of 99 years, the length of the life tables. Benefits are deflated using an assumed yearly interest rate $r = 0.02$. Focussing on the case of a single man at age 65, the expected benefit in period n is the product of the household-specific benefit and the probability of being this household type in period n, ${}_n s_w$. Likewise for a single woman. The case for couples is more complex. Households are a single man with probability ${}_n s_m \cdot (1 - {}_n s_w)$, i.e. the man survived until period n while the woman has died. Benefits are scaled up by $\sqrt{2}$ in case of couples, which happens with probability ${}_n s_m \cdot {}_n s_w$, i.e. the man and woman both survive.

The annuity benefit, i.e. annuity value of assets, is found by solving $A = \mathbb{E}(Benefit(B))$ for B. Because assets vary each year in the data, the benefit B is time-varying within an household.

Household's lifetime income is the average sum of household income and the imputed annuity value of assets. However, the size of the household might change during these years, and differs across households. Then, married couples by definition would have high lifetime income. To tackle this problem, we equivalize household's income with equivalence scale $\sqrt{2}$ so to make couples and singles comparable in terms of their income (cf. Attanasio and Emmerson (2003)). Formally, we calculate lifetime income PI_i of household i as follows:

$$PI_i = \frac{\sum_{\tau=1}^{N_i} B_{i\tau} + \frac{y_{i\tau}}{\sqrt{2}} \cdot marstat_{i\tau} + y_{i\tau} \cdot (1 - marstat_{i\tau})}{N_i},$$

where $y_{i\tau}$ is household income in year τ, $B_{i\tau}$ annuity value of assets, N_i the number of panel observations of household i and $marstat_{i\tau}$ an indicator on whether the household is a couple or single person.

[24]see: `https://opendata.cbs.nl/statline/#/CBS/nl/dataset/37360ned/table?fromstatweb`

B Simulation procedure

B.1 Log-likelihood estimation of $\lambda_k(t, marstat(t); \nu^k, \boldsymbol{\gamma}_k, \boldsymbol{\beta}_k)$

Suppose we want to estimate the unknown parameters $\boldsymbol{\gamma}_k$, $\boldsymbol{\beta}_k$, and σ_k of the hazard rate $\lambda_k(t, marstat(t); \nu^k, \boldsymbol{\gamma}_k, \boldsymbol{\beta}_k)$, specified in (8). We will apply a log-likelihood estimation procedure to estimate the parameters of transition k. We will derive the probability distribution that is input for the individual log-likelihood contribution (we drop index i). To further save on notation, we drop $\boldsymbol{\gamma}_k$ and $\boldsymbol{\beta}_k$; our examples refer to an individual with a given initial marital status, lifetime income group and gender.

Before we derive the probability distribution of interest, we have to discuss the implications of our competing risk setting. Essentially, two transitions are possible at any age, and one will preclude the other from actually occurring. For example, No long-term care use $\rightarrow$ Death happens at random age $T_D = t^*$ while No long-term care use $\rightarrow$ long-term care use would happen at random age $T_L = t^{**} > t^*$. We want to estimate the distribution (transition rate) of both T_L and T_D. Note that the researcher knows $T_L \geq t^*$ but $T_L = t^{**} > t^*$ is hidden information. The competing risks require a log-likelihood function involving the joint distribution of the observed event: $\mathbb{P}(T_D = t^*, T_L \geq t^* \mid \nu^L, \nu^D)$. This distribution simplifies because we assume random effects to be independent across transitions, i.e., $\nu^D \perp \nu^L$: $\mathbb{P}(T_D = t^*, T_L \geq t^* \mid \nu^L, \nu^D) = \mathbb{P}(T_D = t^* \mid \nu^D) \cdot \mathbb{P}(T_L \geq t^* \mid \nu^L)$. Like the distribution function, the likelihood function will split into two sub-likelihoods and we can estimate the transition rates with separate regressions, each for a transition k. The event time T_L would be modeled as randomly right-censored at t^* ($\mathbb{P}(T_L \geq t^* | \ \nu_L)$).

To explain the estimation of a single transition, we look at an example of an individual with two spells of type k. The first spell starts at age $t_{0,1} > 0$, implying a left-truncated observation, for example, because the individual is older than 65 when entering the sample. The other spell starts at age $t_{0,2} > t_{0,1} > 0$. The spells end at ages $t_1 < t_{0,2}$ and t_2, meaning the first spell ends before the next spell starts. The log-likelihood is based on the joint survival probability of staying in the state until ages t_1 and t_2, given you entered the state at ages $t_{0,1}$ and $t_{0,2}$. As we will show below, the hazard rate (8) fully characterizes the distribution T, the random age at transition.

Besides left-truncation, our estimation also considers that marital status is a time-varying covariate. In the example, we assume that the individual is married during spell 1, i.e. $marstat(t) = 1$ if $t \leq t_1$. The individual becomes widowed during spell 2 at age t_w: $t_{0,2} < t_w < t_2$, so $marstat(t) = 1$ if $t < t_w < t_2$ and $marstat(t) = 0$ if $t > t_w$.

The first ingredient to construct the log-likelihood is to have the integrated hazard rate m_k, i.e. the transition rate on having made a transition between age 0 and t:

$$\begin{aligned}
m_k(t;\nu^k, marstat = x) &= \int_0^t \lambda_k(\tau, marstat = x; \nu^k, \boldsymbol{\gamma}_k, \boldsymbol{\beta}_k) d\tau \\
&= \nu^k \cdot \int_0^t \lambda_k(\tau, marstat = x; \nu^k = 1, \boldsymbol{\gamma}_k, \boldsymbol{\beta}_k) d\tau \\
&= \nu^k \cdot m_k(t; \nu^k = 1, marstat = x),
\end{aligned} \tag{9}$$

where we can go from step 1 to steps 2 and 3 because the hazard rate is proportional in ν^k. The alternative representations using $\lambda_k(\tau, marstat = x; \nu^k = 1, \boldsymbol{\gamma}_k, \boldsymbol{\beta}_k)$ and $m_k(t; \nu^k = 1, marstat = x)$ have a closed-form solution (see: Bender et al., 2005) and make it easier to derive a closed-form solution for the log-likelihood contribution.

The marital status in (9) is assumed to have the fixed value $x = \{0, 1\}$ between age 0 and t, i.e. marital status is time-invariant. The accumulated hazards m_k at the left-truncation points $t = t_{0,1}$ and $t = t_{0,2}$ and end age $t = t_1$ are defined according to (9) because marital status only changes after these ages: $t_w > t_{0,2}$. The definition of accumulated hazard at age t_2, however, differs because marital status changes at $t_w < t_2$:

$$\begin{aligned}
m_k(t_2; \nu^k, \{marstat(s)\}_{s=t_{0,2}}^{t_2}) &= m_k(t_w; \nu^k, marstat = 1) \\
&\quad + m_k(t_2; \nu^k, marstat = 0) - m_k(t_w; \nu^k, marstat = 0)
\end{aligned}$$

where $\{marstat(s)\}_{s=t_{0,2}}^{t_2}$ denotes the covariate path of marital status between age $t_{0,2}$ and t_2. The accumulated hazard consists of the sum of hazard until t_w when married ($marstat = 1$) plus the hazard accumulated between t_w and t_2 when single ($marstat = 0$).

The joint survival probability of not having made the transition until ages t_1 and t_2 is linked to the integrated hazard rates is:

$$\begin{aligned}
&\mathbb{P}_k(T_1 > t_1, T_2 > t_2 \mid \{marstat(s)\}_{s=0}^{t_2}, \nu^k) \\
&= \exp\left(-\left\{m_k(t_1; \nu^k, marstat = 1) + m_k(t_2; \nu^k, \{marstat(s)\}_{s=t_{0,2}}^{t_2})\right\}\right),
\end{aligned}$$

which is the exponential function where the negative sum of accumulated hazards serves as input (see: Bender et al., 2005).

For the left truncation points, we can do the same, i.e. the survival probability of not having made the transition by ages $t_{0,1}$ and $t_{0,2}$:

$$\begin{aligned}
&\mathbb{P}_k(T_1 > t_{0,1}, T_2 > t_{0,2} \mid \nu^k, \{marstat(s)\}_{s=0}^{t_{0,2}}) \\
&= \exp\left(-\left\{m_k\left(t_{0,1}; \nu^k, marstat = 1\right) + m_k\left(t_{0,2}; \nu^k, marstat = 1\right)\right\}\right).
\end{aligned}$$

The log-likelihood contribution is based on the joint survival probability of staying in the state until ages t_1 and t_2, given you entered the state at ages $t_{0,1}$ and $t_{0,2}$:

$$\mathbb{P}_k(T_1 > t_1, T_2 > t_2 \mid T_1 > t_{0,1}, T_2 > t_{0,2}, \cdot, \nu^k) = \frac{\mathbb{P}_k(T_1 > t_1, T_2 > t_2 \mid \cdot, \nu^k)}{\mathbb{P}_k(T_1 > t_{0,1}, T_2 > t_{0,2} \mid \cdot, \nu^k)},$$

where for notational convenience we replace the marital histories by a dot $\cdot$.

Lastly, we back out the random effect ν_k, which we do by integrating over its distribution:

$$\begin{aligned}
&\mathbb{P}_k(T_1 > t_1, T_2 > t_2 \mid T_1 > t_{0,1}, T_2 > t_{0,2}, \cdot) \\
&= \int_0^\infty \frac{\mathbb{P}_k(T_1 > t_1, T_2 > t_2 \mid \nu^k, \cdot)}{\mathbb{P}_k(T_1 > t_{0,1}, T_2 > t_{0,2} \mid \nu^k, \cdot)} \, \mathrm{d}\Gamma(\nu^k \mid T_1 > t_{0,1}, T_2 > t_{0,2}, \cdot) \\
&= \frac{\int_0^\infty \mathbb{P}_k(T_1 > t_1, T_2 > t_2 \mid \nu^k, \cdot) \, \mathrm{d}\Gamma(\nu^k)}{\int_0^\infty \mathbb{P}_k(T_1 > t_{0,1}, T_2 > t_{0,2} \mid \nu^k, \cdot) \, \mathrm{d}\Gamma(\nu^k)} \\
&= \frac{\Big\{\sigma_k^2 \cdot \Big\{m_k(t_1; \nu^k = 1, marstat = 1) + m_k(t_2; \nu^k = 1, \{marstat(s)\}_{s=t_{0,2}}^{t_2})\Big\} + 1\Big\}^{-\frac{1}{\sigma_k^2}}}{\{\sigma_k^2 \cdot \{m_k(t_{0,1}; \nu^k = 1, marstat = 1) + m_k(t_{0,2}; \nu^k = 1, marstat = 1)\} + 1\}^{-\frac{1}{\sigma_k^2}}},
\end{aligned} \tag{10}$$

where the final closed-form expression is the probability distribution we use to construct the individual log-likelihood contribution (for the derivation, see: van der Vaart and van den Berg, 2023). The first step – where we integrate over the conditional distribution of the random effect– reflects dynamic selection. Only a particular share of the initial population survives until these dates, presumably driven by their favorable random effect. Hence, the left-truncated distribution deviates from the initial distribution $\Gamma(\nu^k)$. The second step uses the initial distribution instead, see van den Berg and Drepper (2016) for the justification. The last step arrives at the closed-form solution because m_k analyzed at $\nu^k = 1$ has a closed-form solution itself (see Bender et al. (2005) for the solution of m_k for the Gompertz case).

Note the current case involves right censoring. We here provided the cumulative probability of staying in a state until a particular age. This refers to the case when we stop observing the individual at ages t_1 and t_2 while the actual transition is not yet made, e.g. due to the end of the observational window or realization of a competing risk (right censoring). Instead, the log-likelihood contribution involves a probability density if the individual actually makes the transition. This is done by taking the derivative of the probability distribution (10) with respect to random variable T_1 or T_2 and subsequently multiplying the derivative by -1 (to accommodate that we want a cumulative distribution function, i.e. $<$, instead of a survival function, i.e. $\geq$ probabilities). van der Vaart and van den Berg (2023) provide the log-likelihood contribution for a general case of n spells of an individual.

A final remark involves the value of the log-likelihood function. The survival probability (10) involves only transition k but not its competing risk, hence the accompanying log-likelihood is a sub-log-likelihood, particular for transition k. If we add the log-likelihood for the competing risk to this, we obtain the overall likelihood that we effectively maximize. As said, the two sub-log-likelihoods can be optimized separately because the unobservable (random) effect is assumed to be uncorrelated across transitions.

We refer to (Honoré, 1993) and (van den Berg, 2001) and the references therein for parameter identification.

B.2 Simulation

We use estimates for the hazard rates of (8) and (9) to the simulate lifetime duration of long-term care use and the timing of death for 100,000 households. Households initially consist a couple of two members or a single member aged 65 years old. Denote the age of entering the current state by t_0, where $t_0 = 0$ means entry at age 65. We are interested in the subsequent state (not using long-term care, using long-term care, or death) and at what random age $T > t_0$ this transition occurs. We repeat looking for the next state until every individual has died. Finally, we have for each individual a sequence of consecutive states and age at which these states start.

With slight abuse of notation, let the estimates for the integrated hazard rates (9) be denoted by $\widehat{m}_k(t; \nu^k = 1, marstat = x) = \widehat{m}_{k,x}(t)$. $\widehat{m}_{k,x}(t)$ refers to an individual with current marital status x who is endowed with a gender, initial marital status, and lifetime income group. Hence, $\widehat{m}_{k,x}(t)$ can differ across individuals. For now we assume x is fixed during life, i.e. we assume initially married individuals to be currently married and assume that they stay married until they die ($x = 1$). Initial singles remain unmarried throughout ($x = 0$). We introduce widowhood later.

Timing of transition k We use $\widehat{m}_{k,x}(t)$ to compute when a transition of type k, e.g. no long-term care use $\rightarrow$ death, would take place. To this end, we draw a transition time from a conditional survival probability like (10): Given that the individual entered the state at age $T > t_0$, the transition k does not occur before age $T > t > t_0$. This gives: [25]

$$\mathbb{P}_k(t \mid t_0, x) = \mathbb{P}(T > t \mid T > t_0, x, k \text{ occurs}) = \frac{\left(\widehat{\sigma_k^2} \cdot \widehat{m}_{k,x}(t) + 1\right)^{-\frac{1}{\widehat{\sigma_k^2}}}}{\left(\widehat{\sigma_k^2} \cdot \widehat{m}_{k,x}(t_0) + 1\right)^{-\frac{1}{\widehat{\sigma_k^2}}}} \sim \mathcal{U}(0,1)$$

[25] Alternatively, we endowed individuals with an individual-specific effect according to $\widehat{\Gamma}_k$ and subsequently simulated their long-term care use and mortality. Our current approach fits age-specific mortality rates and long-term care use rates better.

Related to our case, Bender et al. (2005) provide the closed-form solution of $\widehat{m}_{k,x}(t)$ when the baseline hazard is of Gompertz form.

The key to the simulation is that survival probability $\mathbb{P}_k(t \mid t_0, x)$ is uniformly distributed itself. Suppose we randomly generate $u \in U(0,1)$ and let $\mathbb{P}_k(t \mid t_0, x) = u$. The value t for which the equation holds, is a randomly generated age t_k at which transition k occurs:

$$t_k = \{\widehat{m}_{k,x}\}^{-1}(\underline{t}), \text{ with: } \underline{t} = \frac{1}{\widehat{\sigma_k^2}} \cdot \left\{ u^{-\widehat{\sigma_k^2}} \cdot \left\{ \widehat{\sigma_k^2} \cdot \widehat{m}_{k,x}(t_0) + 1 \right\} - 1 \right\}.$$

Hence, we have a closed-form solution to simulate age t_k when transition k would occur.

Our simulation considers that other transitions are possible, i.e. 'not using long-term car $\rightarrow$ using long-term care', that might preclude the transition 'not using long-term care $\rightarrow$ death' from occurring. We generate a random age t_k for each possible transition. The minimum across these ages defines the next state and the value for t_0 with which we continue the simulation. We end the simulation if the next state is death.

Widowhood So far we assumed that initially married individuals remain married until death. However, one of the two couple members will die first, and the surviving household member becomes single. Becoming single affects the hazard rate $\widehat{m}_{k,x}$ and thereby thus the timing of a transition. While transition paths before widowhood remain unchanged, we modify the simulated transitions for surviving partner after he or she has become widowed. Remarriage after widowhood is not possible.

For this, we distinguish two types of transitions. First, we look at the transition that is the first to occur after widowhood time t_w. If the individual remained married, the transition would take place at simulated age $t_{k,\text{orig}}$. The individual's accumulated hazard is $\widehat{m}_{k,x=1}(t_{k,\text{orig}})$, which is a counterfactual. The true accumulated hazard is the accumulated hazard until widowhood $\widehat{m}_{k,x=1}(t_w)$ complemented with the hazard while being single: $\widehat{m}_{k,x=0}(t_k) - \widehat{m}_{k,x=0}(t_w)$. To incorporate a widowhood effect to t_k, we set the counterfactual and true hazard equal and solve for t_k:

$$\begin{aligned} &\widehat{m}_{k,x=1}(t_{k,\text{orig}}) = \widehat{m}_{k,x=1}(t_w) + \widehat{m}_{k,x=0}(t_k) - \widehat{m}_{k,x=0}(t_w) \rightarrow \\ &\qquad t_k = \{\widehat{m}_{k,x=0}\}^{-1}(\underline{t}), \text{ with: } \underline{t} = \widehat{m}_{k,x=1}(t_{k,\text{orig}}) - \widehat{m}_{k,x=1}(t_w) + \widehat{m}_{k,x=0}(t_w). \end{aligned}$$

Like earlier, the minimum age across possible transitions determines the next state.

All spells that start after widowhood ($t_0 > t_w$) have a survivor probability as follows:

$$\frac{\left(\widehat{\sigma_k^2} \cdot \{\widehat{m}_{k,x=1}(t_w) + \widehat{m}_{k,x=0}(t) - \widehat{m}_{k,x=0}(t_w)\} + 1\right)^{-\frac{1}{\widehat{\sigma_k^2}}}}{\left(\widehat{\sigma_k^2} \cdot \{\widehat{m}_{k,x=1}(t_w) + \widehat{m}_{k,x=0}(t_0) - \widehat{m}_{k,x=0}(t_w)\} + 1\right)^{-\frac{1}{\widehat{\sigma_k^2}}}} \sim \mathcal{U}(0,1),$$

and the simulated age at transition is:

$$t_k = \{\widehat{m}_{k,x=0}\}^{-1}(\underline{t}), \text{ with:}$$
$$\underline{t} = \frac{1}{\widehat{\sigma_k^2}} \cdot \{u^{-\widehat{\sigma_k^2}} \cdot \{(\widehat{\sigma_k^2} \cdot \{\widehat{m}_{k,x=0}(t_0) - \widehat{m}_{k,x=0}(t_w) + \widehat{m}_{k,x=1}(t_w)\} + 1\} - 1\} + \widehat{m}_{k,x=0}(t_w) - \widehat{m}_{k,x=1}(t_w).$$

Initialization We endow households with initial marital status, long-term care use, and lifetime income according to the empirical distribution of households when the members are aged 65. Sample sizes are provided in Table 4.

Table 4: Initial household distribution in simulation ($N = 100,000$)

	No LTC	**Man in LTC**	**Woman in LTC**	**Both in LTC**	**All**[2]	**Share** (%)
All	96,193	1,716	2,047	44	100,000	100.0
Couple						
Bottom Lifetime IQ	3,399	61	71	12	3,543	6.3
2nd Lifetime IQ	7,285	79	102	12	7,478	13.2
3rd Lifetime IQ	12,004	96	121	11	12,232	21.6
4th Lifetime IQ	15,046	92	99	6	15,243	27.0
Top Lifetime IQ	17,868	87	89	3	18,047	31.9
All	55,602	415	482	44	56,543	100.0
Single men						
Bottom Lifetime IQ	2,324	435			2,759	17.4
2nd Lifetime IQ	1,731	327			2,058	13.0
3rd Lifetime IQ	2,505	260			2,765	17.5
4th Lifetime IQ	3,666	177			3,843	24.3
Top Lifetime IQ	4,297	102			4,399	27.8
	14,523	1,301			15,824	100.0
Single women						
Bottom Lifetime IQ	6,530		751		7,281	26.3
2nd Lifetime IQ	3,938		337		4,275	15.5
3rd Lifetime IQ	4,615		223		4,838	17.5
4th Lifetime IQ	5,542		162		5,704	20.6
Top Lifetime IQ	5,443		92		5,535	20.0
All	26,068		1,565		27,633	100.0

This table shows the sample distribution at age 65. Long-term care use is measured when the household member is aged 65, also when there is an age difference between couple members.

IQ = Income Quintile

[1]The share of an income quintile is not exactly 20% because the lifetime income distribution is determined by all households instead of only those who were aged 65 during the sampling period.

[2] The total number of simulated households is 100,000, for which we provide the counts in this table. The actual number of households in the data was higher.

C Demand curves, WTPs, and comparative statics

First, we derive the incentive compatibility constraints, demand curves and willingness-to-pay (WTP) for buying an annuity, LTC insurance, respectively. We have the utility function:

$$V() = U(C_1) + (s(\xi) - l(\xi))U(C_2^h) + l(\xi)U(C_2^l)$$

Let $L = 1$ if consumer buys a LTC insurance; =0 otherwise; $A = 1$ if consumer buys annuity insurance, 0 otherwise. We assume no saving, so $C_1 = W_1 - P_A A - P_L L$; $C_2^H = W_2 + Y \cdot A$; $C_2^L = W_2 + Y \cdot A - X \cdot (1 - L)$. Substitution of these equalities yields the following direct utility function:

$$V(A, L; W_1, W_2, X, Y, P_L, P_A, \xi) = U(W_1 - P_A A - P_L L) + (s(\xi) - l(\xi))U(W_2 + Y \cdot A) + l(\xi)U(W_2 + Y \cdot A - X \cdot (1 - L))$$

A consumer buys LTC insurance if:

$$V(A^*, 1; W_1, W_2, X, Y, P_L, P_A, \xi) - V(A^*, 0; W_1, W_2, X, Y, P_L, P_A, \xi) \geq 0,$$

i.e. the utility when insured exceeds that of being uninsured. Demand for LTC insurance $D_L(P_L|A^*, W_1, W_2, X, Y, P_A)$ is the likelihood that this inequality holds:

$$\begin{aligned} D_L(P_L|\cdot) &= \mathbb{P}(-(U(W_1 - P_A A^*) - U(W_1 - P_A A^* - P_L)) \\ &\quad + l(\xi)(U(W_2 + Y A^*) - U(W_2 + Y A^* - X)) > 0) \\ &= \mathbb{P}(IC_L(A^*, W_1, W_2, X, Y, P_L, P_A, \xi) > 0), \end{aligned} \tag{11}$$

where IC_L is short-hand notation for the left hand side of the incentive compatibility constraint for LTC insurance. Notice that IC_L (a utility difference) is strictly decreasing in P_L. Therefore, demand $D_L(P_L|\cdot)$ will be strictly decreasing in P_L. We can meaningfully define the WTP as follows:

$$\pi_L(A^*, W_1, W_2, X, Y, P_A, \xi) = \max\{P_L; IC_L(A^*, W_1, W_2, X, Y, P_L, P_A, \xi) \leq 0)\}$$

$\pi_L(\cdot)$ can be solved from the following implicit equation:

$$IC_L(A^*, W_1, W_2, X, Y, \pi_L(\cdot), P_A, \xi) = 0$$

A consumer buys stand alone annuity insurance if $V(1, L^*, W_1, W_2, X, Y, P_L, P_A, \xi) - V(0, L^*, W_1, W_2, X, Y, P_L, P_A, \xi) \geq 0$. The demand curve is the probability that this incentive compatibility constraint holds:

$$\begin{aligned} D_A(P_A|L^*, W_1, W_2, X, Y, P_L) &= \mathbb{P}(-(U(W_1 - P_L L^*) - U(W_1 - P_A - P_L L^*)) \\ &\quad + s(\xi)(U(W_2 + Y) - U(W_2)) + (1 - L^*)l(\xi)LL(W_2, Y, X) > 0) \\ &= \mathbb{P}(IC_A(L^*, W_1, W_2, X, Y, P_L, P_A, \xi) > 0) \qquad (12) \end{aligned}$$

with $LL(W_2, Y, X) = ((U(W_2 + Y - X) - U(W_2 + Y)) - (U(W_2 - X) - U(W_2)))$. Since $U()$ is strictly concave and $W_2 > 0$, $Y > 0$ and $X > 0$, $LL(W_2, Y, X) > 0$.

Notice that $IC_A(\cdot)$ in (12) (a utility difference) is strictly decreasing in P_A. This implies that the demand curve is also decreasing in P_A. We can meaningfully define WTP as follows:

$$\pi_A(L^*, W_1, W_2, X, Y, P_L, \xi) = \max\{P_A; IC_A(L^*, W_1, W_2, X, Y, P_L, P_A, \xi) \leq 0\}$$

In other words, $\pi_A(\cdot)$ can be solved from the following implicit equation:

$$IC_A(L^*, W_1, W_2, X, Y, P_L, \pi_A(\cdot), \xi) = 0$$

Lastly, consider the case of a life care annuity. Suppose that stand alone insurances are not available. We assume no saving, so $C_1 = W_1 - P_{CA} \cdot CA$; $C_2^H = W_2 + Y \cdot CA$; $C_2^L = W_2 + Y \cdot CA + (\rho \cdot Y \cdot CA - X)$. Substitution of these equalities yields the following direct utility function:

$$\begin{aligned} V(CA; W_1, W_2, \rho, Y, X, P_{CA}) &= U(W_1 - P_{CA}CA) + (s(\xi) - l(\xi))U(W_2 + Y \cdot CA) \\ &\quad + l(\xi)U(W_2 + Y \cdot CA + (\rho Y \cdot CA - X)) \end{aligned}$$

A consumer buys a life care annuity if $V(1; W_1, W_2, \rho, Y, X, P_{CA}) - V(0; W_1, W_2, \rho, Y, X, P_{CA}) \geq 0$. Then, demand for a life care annuity is given by the probability that this incentive compatibility constraint is met:

$$\begin{gathered} D_{CA}(P_{CA}|W_1, W_2, \rho, Y, X) = \mathbb{P}(-(U(W_1) - U(W_1 - P_{CA})) + s(\xi)(U(W_2 + Y) - U(W_2)) + \\ l(\xi)LLL(W_2, \rho, Y, X) \geq 0) = \mathbb{P}(IC_{CA}(W_1, W_2, \rho, Y, X, P_{CA}, \xi) \geq 0). \end{gathered} \qquad (13)$$

where

$$LLL(W_2, \rho, Y, X) = ((U(W_2 + Y + (\rho Y - X)) - U(W_2 + Y)) - (U(W_2 - X) - U(W_2)))$$

Since $U()$ is strictly concave and $W_2 > 0$, $Y > 0$ and $X > 0$, $LLL(W_2, \rho, Y, X) > 0$.

Notice that $IC_{CA}(\cdot)$ in (13) (a utility difference) is strictly decreasing in P_{CA}. Moreover, $D_{CA}(P_{CA}|\cdot)$ is strictly decreasing in P_{CA}. So, we can meaningfully define the Willingness To Pay (WTP) as follows:

$$\pi_{CA}(W_1, W_2, \rho, Y, X, \xi) = \max\{P_{CA}; IC_{CA}(W_1, W_2, \rho, Y, X, P_{CA}, \xi) = 0\}$$

In other words, $\pi_{CA}(\cdot)$ can be solved from the following implicit equation:

$$IC_{CA}(W_1, W_2, \rho, Y, X, \pi_{CA}(\cdot), \xi) = 0.$$

We now derive the comparative statics of the demand curve considering the premium and correlation between risks l and s. The demand curves (11), (12), and (13) can be written as the following implicit functions:

$$\begin{aligned} D_L(P_L|W_1, W_2, X, Y, P_A) &= \mathbb{P}(l(\xi) \cdot \upsilon_{2,l,L}(A^*, W_2, Y, X) \geq \upsilon_{1,L}(P_L|W_1, P_A, A^*)) \\ D_A(P_A|L^*, W_1, W_2, X, Y, P_L) &= \mathbb{P}(l(\xi) \cdot \upsilon_{2,l,A}(L^*, W_2, Y, X) + s(\xi) \cdot \upsilon_{2,s,A}(W_2, Y) \geq \upsilon_{1,A}(P_A|W_1, P_L, L^*)) \\ D_{CA}(P_{CA}|W_1, W_2, \rho, Y, X) &= \mathbb{P}(l(\xi) \cdot \upsilon_{2,l,CA}(W_2, \rho, Y, X) + s(\xi) \cdot \upsilon_{2,s,CA}(W_2, Y) \geq \upsilon_{1,CA}(P_{CA}|W_1)). \end{aligned}$$

Note that these demand curves are of the form $\mathbb{P}(l \cdot \upsilon_{2,l} + s \cdot \upsilon_{2,s} \geq \upsilon_1)$ where (s, l) are potentially correlated risks and $\upsilon_{2,l} > 0$, $\upsilon_{2,s} > 0$ and $\upsilon_1 \geq 0$ are scalars determining demand. $\upsilon_{2,l}$ and $\upsilon_{2,s}$ are the utility gains from insurance coverage in period 2 of risks l and s, respectively. υ_1 is the utility loss in period 1 due to paying a premium for the insurance. Obviously, the larger the insurance utility gains $\upsilon_{2,l} > 0$ and $\upsilon_{2,s}$ are, the more likely a consumer will buy insurance. Also, the lower the premium, the smaller the utility loss υ_1 is, and hence the more likely the demand for an insurance product is. Formally:

$$\frac{\partial D_K(P_K|\cdot)}{\partial P_K} = \overbrace{\frac{\mathbb{P}(\cdot)}{\partial \upsilon_{1,K}}}^{<0} \cdot \overbrace{\frac{\partial \upsilon_{1,K}}{\partial P_K}}^{>0} < 0$$

which means that demand is lower if the premium is higher ($K \in (L, A, CA)$).

Next, we ask ourselves: does a demand curve of the form $D(P) = \mathbb{P}(l \cdot \upsilon_{2,l} + s \cdot \upsilon_{2,s} \geq \upsilon_1(P))$ becomes *steeper* or *flatter* if we decrease the correlation θ between risks l and s? Put concretely, we are interested in the comparative statics:

$$\frac{\partial^2 D_L(P_L; \theta)}{\partial P_L \partial \theta}; \qquad \frac{\partial^2 D_A(P_A; \theta)}{\partial P_A \partial \theta}; \qquad \frac{\partial^2 D_{CA}(P_{CA}; \theta)}{\partial P_{CA} \partial \theta}.$$

To this end, we have to explicitly derive the demand curve $D(P|\theta) = \mathbb{P}(l \cdot \upsilon_{2,l} + s \cdot \upsilon_{2,s} \geq \upsilon_1|\theta)$ as a function of correlation θ, because that parameter is missing in the current demand function. This requires knowledge of the joint distribution of s and l and its dependence on correlation θ. Also, fixing everything else for our comparative static means that we want to fix the marginal distributions in the population of l and s, and only vary the part of the joint distribution that involves the correlation structure. Define

$\Gamma(F_l, F_s)$ to be the set of joint distribution functions with marginals $F_l = \mathbb{P}(l \leq L)$ and $F_s = \mathbb{P}(s \leq S)$. Following Solomon (2022) the correlation structure of interest is:

Definition of a correlation order *Suppose we have two populations $X, Y \in \Gamma(F_l, F_s)$ and have joint CDFs F_X, F_Y, respectively. Solomon (2022) defines the correlation between l and s in population X is less correlated than in population Y or that X precedes Y in correlation order, written as $X \precsim Y$ if and only if:*

$$\mathbb{P}(s \leq S, l \leq L|X) = F_X(S, L) \leq F_Y(S, L) = \mathbb{P}(s \leq S, l \leq L|Y) \textit{ for all } (S, L) \in D_F,$$

so the probability of a pair with low (s, l) is smaller in population X than in Y, implying the correlation is more negative in population X.

Ideally we have the same marginal distribution in F_l and F_s and modify the joint relationship between the two variables only via a correlation parameter. A class of distribution functions that meet these needs including a correlation order, are those of Farlie-Gumble-Morgenstern form (Denuit and Scaillet, 2004):

$$F(S, L) = F_l(L) \cdot F_s(S) \cdot (1 + \theta \cdot (1 - F_l(L)) \cdot (1 - F_s(S))) \tag{14}$$

with $\theta \in [-1, 1]$ governing the dependence between the two marginals and $\theta = 0$ implying independent distributions for s and l.

For simplicity, we assume $l \sim \mathcal{U}(0, 1)$ and $s \sim \mathcal{U}(0, 1)$ so $F_l(L) = L$ and $F_s(S) = S$. Then

$$\begin{aligned} &F(S, L) = L \cdot S \cdot (1 + \theta \cdot (1 - L) \cdot (1 - S)) \implies \\ &f(S, L) = 1 + \theta \cdot (1 - 2L) \cdot (1 - 2S) \end{aligned}$$

To find the demand functions, we have to find the convolution: $\mathbb{P}(l \cdot v_{2,l} + s \cdot v_{2,s} \geq v_1|\theta) = 1 - \mathbb{P}(l \cdot v_{2,l} + s \cdot v_{2,s} \leq v_1|\theta)$. We derived the closed form solutions $\mathbb{P}(l \cdot v_{2,l} + s \cdot v_{2,s} \leq v_1|\theta)$, which are:

$$\begin{array}{lr}
\frac{v_1^2}{2v_{2,l}v_{2,s}} + \theta \cdot \frac{1}{6} \cdot \frac{v_1^2}{v_{2,s}^4} \cdot (v_1^2 - 2(v_{2,l} + v_{2,s}) \cdot v_1 + 3v_{2,l}v_{2,s}), & \text{if } v_1(P) \leq \min(v_{2,l}, v_{2,s}) \\
\frac{v_1}{v_{2,s}} - \frac{v_{2,l}}{2 \cdot v_{2,s}} + \theta \cdot \frac{1}{6} \cdot \frac{v_{2,l}}{v_{2,s}} \cdot \left(1 - 2 \cdot \left(\frac{v_1 - v_{2,l}}{v_{2,s}}\right) - \frac{v_{2,l}}{v_{2,s}}\right), & \text{if } v_1(P) \in [v_{2,l}, v_{2,s}] \\
\frac{v_1}{v_{2,l}} - \frac{v_{2,s}}{2 \cdot v_{2,l}} + \theta \cdot \frac{1}{6} \cdot \frac{v_{2,s}}{v_{2,l}} \cdot \left(1 - 2 \cdot \left(\frac{v_1 - v_{2,s}}{v_{2,l}}\right) - \frac{v_{2,s}}{v_{2,l}}\right). & \text{if } v_1(P) \in [v_{2,s}, v_{2,l}] \\
1 - \frac{v_{2,l} + \frac{1}{2}v_{2,s} - v_1}{v_{2,l}} - \frac{1}{2} \cdot \frac{(v_1 - v_{2,l})^2}{v_{2,l}v_{2,s}} - \theta \cdot \frac{1}{6} \cdot \frac{v_{2,s}}{v_{2,l}} \cdot & \\
\left(\frac{v_1 - v_{2,s}}{v_{2,l}} + \frac{v_1 - v_{2,l}}{v_{2,l}} - 3 \cdot \left(\frac{v_1 - v_{2,l}}{v_{2,s}}\right)^2\right) & \\
-\theta \cdot \frac{1}{6} \cdot \frac{v_{2,s}}{v_{2,l}} \cdot \left(2 \cdot \left(1 - \frac{v_{2,s}}{v_{2,l}}\right) \cdot \left(\frac{v_1 - v_{2,l}}{v_{2,s}}\right)^3 + \frac{v_{2,s}}{v_{2,l}} \cdot \left(\frac{v_1 - v_{2,l}}{v_{2,s}}\right)^4\right), & \text{if } v_1(P) \geq \max(v_{2,l}, v_{2,s}),
\end{array}$$

which depends on the premium level (P) via $\upsilon_1(P)$, with $\upsilon_1'(P) > 0$.

We are interested in the sign of the comparative static:

$$\frac{\partial^2 \mathbb{P}(l \cdot \upsilon_{2,l} + s \cdot \upsilon_{2,s} \geq \upsilon_1(P)|\theta)}{\partial P \partial \theta} = -\frac{\partial^2 \mathbb{P}(l \cdot \upsilon_{2,l} + s \cdot \upsilon_{2,s} \leq \upsilon_1(P)|\theta)}{\partial P \partial \theta} = \upsilon_1'(P) \cdot - \frac{\partial^2 \mathbb{P}(l \cdot \upsilon_{2,l} + s \cdot \upsilon_{2,s} \leq \upsilon_1|\theta)}{\partial \upsilon_1 \partial \theta}$$

The relevant part of the comparative static is $-\frac{\partial^2 \mathbb{P}(l \cdot \upsilon_{2,l} + s \cdot \upsilon_{2,s} \leq \upsilon_1|\theta)}{\partial \upsilon_1 \partial \theta}$, given by:

$$\begin{cases} -\frac{1}{6} \cdot \frac{\upsilon_1}{\upsilon_{2,s}^4} \cdot (4\upsilon_1^2 - 6(\upsilon_{2,l} + \upsilon_{2,s}) \cdot \upsilon_1 + 6\upsilon_{2,l}\upsilon_{2,s}), & \text{if } \upsilon_1(P) \leq \min(\upsilon_{2,l}, \upsilon_{2,s}) \\ \frac{1}{6} \cdot \frac{\upsilon_{2,l}}{\upsilon_{2,s}} \cdot \frac{2}{\upsilon_{2,s}}, & \text{if } \upsilon_1(P) \in [\upsilon_{2,l}, \upsilon_{2,s}] \\ \frac{1}{6} \cdot \frac{\upsilon_{2,s}}{\upsilon_{2,l}} \cdot \frac{2}{\upsilon_{2,l}}. & \text{if } \upsilon_1(P) \in [\upsilon_{2,s}, \upsilon_{2,l}] \\ \frac{1}{6} \cdot \frac{\upsilon_{2,s}}{\upsilon_{2,l}} \cdot \frac{1}{\upsilon_{2,l}\upsilon_{2,s}^3} \cdot \left(2\upsilon_{2,s}^3 - 6\upsilon_{2,l}\upsilon_{2,s} \cdot (\upsilon_1 - \upsilon_{2,l})\right) & \\ +\frac{1}{6} \cdot \frac{\upsilon_{2,s}}{\upsilon_{2,l}} \cdot \frac{1}{\upsilon_{2,l}\upsilon_{2,s}^3} \cdot \left(6(\upsilon_{2,l} - \upsilon_{2,s}) \cdot (\upsilon_1 - \upsilon_{2,l})^2 + 4 \cdot (\upsilon_1 - \upsilon_{2,l})^3\right), & \text{if } \upsilon_1(P) \geq \max(\upsilon_{2,l}, \upsilon_{2,s}) \end{cases}$$

which has sign:

$$\begin{cases} \leq 0, & \text{if } \upsilon_1(P) \leq \min\left(\frac{3}{4} \cdot \left(\upsilon_{2,l} + \upsilon_{2,s} - \sqrt{\upsilon_{2,l}^2 + \upsilon_{2,s}^2 - \frac{2}{3}\upsilon_{2,l}\upsilon_{2,s}}\right), \upsilon_{2,l}, \upsilon_{2,s}\right) \\ \geq 0, & \text{if } \upsilon_1(P) \in \left[\frac{3}{4} \cdot \left(\upsilon_{2,l} + \upsilon_{2,s} - \sqrt{\upsilon_{2,l}^2 + \upsilon_{2,s}^2 - \frac{2}{3}\upsilon_{2,l}\upsilon_{2,s}}\right), \min(\upsilon_{2,l}, \upsilon_{2,s})\right] \\ \geq 0, & \text{if } \upsilon_1(P) \in [\upsilon_{2,l}, \upsilon_{2,s}] \\ \geq 0, & \text{if } \upsilon_1(P) \in [\upsilon_{2,s}, \upsilon_{2,l}] \\ \geq 0, & \text{if } \upsilon_1(P) \in \left[\max(\upsilon_{2,l}, \upsilon_{2,s}), \frac{3}{4} \cdot \left(\upsilon_{2,l} + \upsilon_{2,s} + \sqrt{\upsilon_{2,l}^2 + \upsilon_{2,s}^2 - \frac{2}{3}\upsilon_{2,l}\upsilon_{2,s}}\right)\right] \\ \leq 0 & \text{if } \upsilon_1(P) \in \left[\frac{3}{4} \cdot \left(\upsilon_{2,l} + \upsilon_{2,s} + \sqrt{\upsilon_{2,l}^2 + \upsilon_{2,s}^2 - \frac{2}{3}\upsilon_{2,l}\upsilon_{2,s}}\right), \upsilon_{2,l} + \upsilon_{2,s}\right], \end{cases}$$

implying that the impact of θ on the slope of the demand curve changes sign maximally twice. If $\Delta\theta < 0$, then the demand function features a *steeper* decline at high values of P, is *flatter* at intermediate values of P, and has a *steeper* decline at low values of P. The demand curve becomes flatter, because total risk exposure is more homogenous if the correlation is more negative, i.e. θ is lower. However, at high and low premia, there is a steeper decline because extreme risk individuals are still possible, i.e. with a high pair of risks $(s = 1, l = 1)$ or a low pair of risks $(s = 0, l = 0)$.

D Derivation of Equation (6)

$$\text{The first-order condition:} \quad 0 = \frac{\partial \mathcal{F}(\rho)}{\partial \rho} = 2 \cdot \mathbb{E}\left(PR(\xi,\rho) \cdot \frac{\partial PR(\xi,\rho)}{\partial \rho}\right), \text{ with:}$$

$$PR(\xi,\rho) = \frac{s(\xi) + l(\xi)\cdot\rho}{\mathbb{E}(s(\xi)) + \mathbb{E}(l(\xi))\cdot\rho} - 1, \tag{15}$$

First, we determine $\frac{\partial PR(\xi,\rho)}{\partial \rho}$. Suppose $\rho \neq -\frac{\mathbb{E}(s(\xi))}{\mathbb{E}(l(\xi))}$, $\mathbb{E}(s(\xi)) \neq 0$ and $\mathbb{E}(l(\xi)) \neq 0$, then:

$$\begin{aligned}
\frac{\partial PR(\xi,\rho)}{\partial \rho} &= \frac{l(\xi)\cdot\{\mathbb{E}(s(\xi)) + \rho\cdot\mathbb{E}(l(\xi))\} - \cdot\mathbb{E}(l(\xi))\cdot\{s(\xi)+\rho\cdot l(\xi)\}}{\{\mathbb{E}(s(\xi)) + \rho\cdot\mathbb{E}(l(\xi))\}^2} \\
&= \frac{l(\xi)\cdot\mathbb{E}(s(\xi)) - \mathbb{E}(l(\xi))\cdot s(\xi)}{\{\mathbb{E}(s(\xi)) + \rho\cdot\mathbb{E}(l(\xi))\}^2} \\
&= \omega(\rho)^2 \cdot \left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))} - \frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\} \text{ and: } \omega(\rho) = \frac{\sqrt{\mathbb{E}(l(\xi))\cdot\mathbb{E}(s(\xi))}}{\mathbb{E}(s(\xi)) + \rho\cdot\mathbb{E}(l(\xi))} \neq 0.
\end{aligned} \tag{16}$$

We use this result to solve first-order condition (15):

$$\begin{aligned}
0 &= 2\cdot\mathbb{E}\left(PR(\xi,\rho)\cdot\left.\frac{\partial PR(\xi,\rho)}{\partial\rho}\right|_{\rho=\rho^\star}\right) \\
&= 2\cdot\mathbb{E}\left(\left\{\frac{s(\xi)-\mathbb{E}(s(\xi)) + \rho^\star\cdot(l(\xi)-\mathbb{E}(l(\xi)))}{\mathbb{E}(s(\xi)) + \rho^\star\cdot\mathbb{E}(l(\xi))}\right\}\cdot\Omega(\xi)\right)\cdot\omega(\rho^\star)^2 \rightarrow \\
&\mathbb{E}\left\{(s(\xi)-\mathbb{E}(s(\xi)))\cdot\Omega(\xi)\right\} = -\rho^\star\cdot\mathbb{E}\left\{(l(\xi)-\mathbb{E}(l(\xi)))\cdot\Omega(\xi)\right\} \rightarrow \\
&\rho^\star = \frac{\mathbb{E}(s(\xi))}{\mathbb{E}(l(\xi))}\cdot\frac{\mathbb{E}\left\{\left(\frac{s(\xi)}{\mathbb{E}(s(\xi))}-1\right)\cdot\Omega(\xi)\right\}}{\mathbb{E}\left\{\left(\frac{l(\xi)}{\mathbb{E}(l(\xi))}-1\right)\cdot -\Omega(\xi)\right\}}, \quad \text{with: } \Omega(\xi) = \frac{l(\xi)}{\mathbb{E}(l(\xi))} - \frac{s(\xi)}{\mathbb{E}(s(\xi))}
\end{aligned}$$

Substituting back $\Omega(\xi)$ gives:

$$\begin{aligned}
\rho^\star &= \frac{\mathbb{E}(s(\xi))}{\mathbb{E}(l(\xi))}\cdot\frac{\mathbb{E}\left\{\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}-1\right\}\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}-\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\}\right\}}{\mathbb{E}\left\{\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}-1\right\}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}-\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}\right\}} \\
&= \frac{\mathbb{E}(s(\xi))}{\mathbb{E}(l(\xi))}\cdot\frac{\mathbb{E}\left\{\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\}\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}\right\} + \mathbb{E}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\} - \mathbb{E}\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\} - \mathbb{E}\left\{\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\}^2\right\}}{\mathbb{E}\left\{\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\}\right\} + \mathbb{E}\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\} - \mathbb{E}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\} - \mathbb{E}\left\{\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}^2\right\}} \\
&= \frac{\mathbb{E}(s(\xi))}{\mathbb{E}(l(\xi))}\cdot\frac{\operatorname{Cov}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))},\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\} - \operatorname{Var}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\}}{\operatorname{Cov}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))},\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\} - \operatorname{Var}\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}} \\
&= \frac{\mathbb{E}(s(\xi))}{\mathbb{E}(l(\xi))}\cdot\frac{\frac{\operatorname{SD}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\}}{\operatorname{SD}\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}} - \operatorname{Corr}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))},\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}}{\left\{\frac{\operatorname{SD}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\}}{\operatorname{SD}\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}}\right\}^{-1} - \operatorname{Corr}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))},\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}}. \quad \square
\end{aligned}$$

Note that to get from step two to three we use the identity $\mathbb{E}\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\} = \mathbb{E}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\} = 1$.

To examine the behavior of $\rho^\star$ to changes in $\frac{\mathbb{E}(s(\xi))}{\mathbb{E}(l(\xi))}$, $\frac{\text{SD}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\}}{\text{SD}\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}}$ and $\text{Corr}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}, \frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}$, we can compute the corresponding partial derivatives:

$$\frac{\partial \rho^\star}{\partial \frac{\mathbb{E}(s(\xi))}{\mathbb{E}(l(\xi))}} = \frac{\mathbb{E}(l(\xi))}{\mathbb{E}(s(\xi))} \cdot \rho^\star = \begin{cases} = 0 \text{ if } \frac{\text{SD}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\}}{\text{SD}\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}} = \text{Corr}\{\cdot\} \\ > 0 \text{ if } \quad \text{Corr}\{\cdot\} \leq 0 \quad \vee \quad \left\{ \text{Corr}\{\cdot\} > 0 \quad \wedge \quad \frac{\text{SD}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\}}{\text{SD}\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}} \in \left(\text{Corr}\{\cdot\}, \ \frac{1}{\text{Corr}\{\cdot\}}\right)\right\} \\ < 0 \text{ elsewhere.} \end{cases}$$

Note $\frac{\partial \rho^\star}{\partial \frac{\text{SD}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\}}{\text{SD}\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}}} = \frac{\mathbb{E}(s(\xi))}{\mathbb{E}(l(\xi))} \cdot \frac{2 \cdot \frac{\text{SD}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\}}{\text{SD}\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}} - \left(\frac{\text{SD}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\}}{\text{SD}\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}}\right)^2 \cdot \text{Corr}\{\cdot\} - \text{Corr}\{\cdot\}}{\left(1 - \frac{\text{SD}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\}}{\text{SD}\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}} \cdot \text{Corr}\{\cdot\}\right)^2}$, then:

$$\frac{\partial \rho^\star}{\partial \frac{\text{SD}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\}}{\text{SD}\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}}} = \begin{cases} = 0 \text{ if } \quad \text{Corr}\{\cdot\} > 0 \quad \wedge \quad \frac{\text{SD}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\}}{\text{SD}\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}} = \frac{1}{\text{Corr}\{\cdot\}} \pm \sqrt{\frac{1}{\text{Corr}\{\cdot\}^2} - 1} \\ < 0 \text{ if } \quad \text{Corr}\{\cdot\} > 0 \quad \wedge \quad \frac{\text{SD}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\}}{\text{SD}\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}} \notin \left(\frac{1}{\text{Corr}\{\cdot\}} - \sqrt{\frac{1}{\text{Corr}\{\cdot\}^2} - 1}, \ \frac{1}{\text{Corr}\{\cdot\}} + \sqrt{\frac{1}{\text{Corr}\{\cdot\}^2} - 1}\right) \\ > 0 \text{ elsewhere.} \end{cases}$$

Lastly:

$$\frac{\partial \rho^\star}{\partial \text{Corr}\{\cdot\}} = \frac{\mathbb{E}(s(\xi))}{\mathbb{E}(l(\xi))} \cdot \frac{\frac{\text{SD}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\}}{\text{SD}\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}} - \left(\frac{\text{SD}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\}}{\text{SD}\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}}\right)^{-1}}{\left(\left\{\frac{\text{SD}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\}}{\text{SD}\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}}\right\}^{-1} - \text{Corr}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}, \frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}\right)^2} \begin{cases} = 0 \text{ if } \frac{\text{SD}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\}}{\text{SD}\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}} = 1 \\ < 0 \text{ if } \frac{\text{SD}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\}}{\text{SD}\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}} < 1 \\ > 0 \text{ if } \frac{\text{SD}\left\{\frac{s(\xi)}{\mathbb{E}(s(\xi))}\right\}}{\text{SD}\left\{\frac{l(\xi)}{\mathbb{E}(l(\xi))}\right\}} > 1 \end{cases}$$

E Descriptive Statistics

Table 5 provides descriptive statistics for our main variables.

Table 5: Descriptive statistics

	Mean	**Median**	**S.D.**	**Min**[3]	**Max**
Individuals (*N = 3,278,797*)					
Uses long-term care (LTC) (%)	39.8				
Passes away (%)	25.3				
Recovers from LTC[1,2] (%)	53.8				
Has multiple LTC spells[1] (%)	24.5				
Number of LTC spells[1]	1.3				
Observed duration LTC [1]	2.3	1.3			
Married households (Unbalanced panel; *Panel observations= 5,906,251*)					
Household income[4] (*000s euros*)					
Bottom Lifetime IQ	21.3	21.2	2.2	2.8	41.4
2nd Lifetime IQ	25.8	25.8	2.9	5.7	57.8
3rd Lifetime IQ	31.4	31.4	4.7	6.8	86.1
4th Lifetime IQ	41.1	41.2	7.8	5.9	120.7
Top Lifetime IQ	67.8	61.4	31.8	6.2	1,038.5
All	40.0	33.4	23.2	2.3	1,038.5
Liquid assets (*000s euros)*					
Bottom Lifetime IQ	16.9	8.9	22.3	0.0	336.8
2nd Lifetime IQ	33.6	22.6	39.0	0.0	628.5
3rd Lifetime IQ	53.7	31.3	64.8	0.0	1,056.3
4th Lifetime IQ	85.2	47.6	103.1	0.0	1,753.7
Top Lifetime IQ	396.5	142.3	3,348.3	0.0	156,145.9
All	133.5	37.2	1,630.6	0.0	156,145.9
Single-person households (Unbalanced panel; *Panel observations= 8,073,927)*					
Women (%)	76.6		42.3		
Household income (*000s euros)*					
Bottom Lifetime IQ	14.9	14.6	1.6	0.1	35.4
2nd Lifetime IQ	17.9	18.0	2.1	0.7	51.0
3rd Lifetime IQ	21.8	22.1	3.7	0.7	81.8
4th Lifetime IQ	28.0	28.5	6.3	0.8	113.1
Top Lifetime IQ	43.6	40.7	21.7	0.7	926.7
All	23.9	19.6	13.5	0.1	926.7
Liquid assets (*000s euros)*					
Bottom Lifetime IQ	10.6	5.1	14.5	0.0	342.5
2nd Lifetime IQ	25.2	16.5	31.3	0.0	589.0
3rd Lifetime IQ	43.6	24.1	55.7	0.0	935.1
4th Lifetime IQ	74.1	38.3	91.4	0.0	1,638.7
Top Lifetime IQ	330.4	131.8	1,628.4	0.0	101,383.2
All	83.4	21.4	668.5	0.0	101,383.2

IQ = Income Quintile

[1]Conditional upon using long-term care.

[2]Recovery: realised transition from LTC to No LTC instead of passing away in LTC.

[3]: Maximum and minimum are the averages of the one hundred highest and lowest values.

[4]: Income in 2015 prices. Savings and bonds in 2015 prices, stocks inflated with AEX stock-index of 31^{st} of December 2014.

Table 6: Hazard rate estimates

Transition	No LTC → LTC	No LTC → Death	LTC→No LTC	LTC → Death
Constant (β_k)	-2.938***	-4.564***	-1.626***	-2.498***
	(0.014)	(0.033)	(0.018)	(0.022)
Single at baseline (β_{1kh})				
Men - Bottom Lifetime IQ	0	0	0	0
	(.)	(.)	(.)	(.)
Men - 2nd Lifetime IQ	0.008	-0.142***	0.044	-0.031
	(0.021)	(0.052)	(0.027)	(0.035)
Men - 3rd Lifetime IQ	-0.239***	-0.192***	0.511***	0.303***
	(0.021)	(0.049)	(0.027)	(0.035)
Men - 4th Lifetime IQ	-0.459***	-0.261***	0.955***	0.684***
	(0.021)	(0.047)	(0.027)	(0.037)
Men - Top Lifetime IQ	-0.798***	-0.624***	1.249***	0.973***
	(0.021)	(0.048)	(0.027)	(0.040)
Women - Bottom Lifetime IQ	-0.027*	-1.245***	0.874***	-0.365***
	(0.015)	(0.041)	(0.020)	(0.026)
Women - 2nd Lifetime IQ	-0.178***	-1.618***	1.203***	-0.317***
	(0.016)	(0.049)	(0.021)	(0.030)
Women - 3rd Lifetime IQ	-0.360***	-1.615***	1.501***	-0.024
	(0.017)	(0.050)	(0.021)	(0.033)
Women - 4th Lifetime IQ	-0.535***	-1.709***	1.754***	0.227***
	(0.017)	(0.051)	(0.022)	(0.035)
Women - Top Lifetime IQ	-0.723***	-1.988***	1.952***	0.499***
	(0.018)	(0.055)	(0.023)	(0.038)
Married at baseline - currently single (β^k_{1kh})				
Men - Bottom Lifetime IQ	-0.442***	-0.000	1.066***	0.952***
	(0.019)	(0.049)	(0.026)	(0.034)
Men - 2nd Lifetime IQ	-0.696***	-0.362***	1.245***	1.075***
	(0.018)	(0.045)	(0.024)	(0.032)
Men - 3rd Lifetime IQ	-0.868***	-0.593***	1.351***	1.175***
	(0.018)	(0.045)	(0.024)	(0.032)
Men - 4th Lifetime IQ	-1.074***	-0.662***	1.477***	1.273***
	(0.018)	(0.045)	(0.024)	(0.034)
Men - Top Lifetime IQ	-1.326***	-0.932***	1.602***	1.382***
	(0.019)	(0.045)	(0.025)	(0.037)
Women - Bottom Lifetime IQ	-0.203***	-1.724***	1.459***	-0.648***
	(0.017)	(0.049)	(0.022)	(0.029)
Women - 2nd Lifetime IQ	-0.411***	-1.731***	1.648***	-0.294***
	(0.016)	(0.047)	(0.021)	(0.029)
Women - 3rd Lifetime IQ	-0.643***	-1.672***	1.773***	0.111***
	(0.017)	(0.047)	(0.021)	(0.030)
Women - 4th Lifetime IQ	-0.864***	-1.863***	1.913***	0.440***
	(0.017)	(0.049)	(0.022)	(0.033)
Women - Top Lifetime IQ	-1.079***	-2.013***	2.000***	0.752***
	(0.018)	(0.050)	(0.023)	(0.035)
Married at baseline - currently single ($\beta^k_{1kh} + \beta^k_{2kh}$)				
Men - Bottom Lifetime IQ	-0.723***	-0.397***	1.546***	1.385***
	(0.016)	(0.038)	(0.022)	(0.029)
Men - 2nd Lifetime IQ	-1.027***	-0.671***	1.727***	1.678***
	(0.016)	(0.037)	(0.021)	(0.028)
Men - 3rd Lifetime IQ	-1.244***	-0.807***	1.821***	1.948***
	(0.016)	(0.036)	(0.021)	(0.028)
Men - 4th Lifetime IQ	-1.448***	-0.958***	1.910***	2.165***
	(0.016)	(0.037)	(0.021)	(0.029)
Men - Top Lifetime IQ	-1.655***	-1.128***	1.949***	2.333***
	(0.016)	(0.037)	(0.022)	(0.030)
Women - Bottom Lifetime IQ	-0.494***	-1.969***	1.738***	-0.362***
	(0.015)	(0.043)	(0.020)	(0.028)
Women - 2nd Lifetime IQ	-0.743***	-1.890***	1.901***	0.091***
	(0.015)	(0.040)	(0.020)	(0.027)
Women - 3rd Lifetime IQ	-0.992***	-1.729***	2.014***	0.688***
	(0.015)	(0.040)	(0.020)	(0.028)
Women - 4th Lifetime IQ	-1.231***	-1.808***	2.115***	1.117***
	(0.016)	(0.040)	(0.020)	(0.029)
Women - Top Lifetime IQ	-1.422***	-1.863***	2.189***	1.617***
	(0.016)	(0.040)	(0.021)	(0.030)

Table 6: (continued)

Transition	No LTC → LTC	No LTC → Death	LTC→No LTC	LTC → Death
γ_k	0.076***	0.137***	-0.028***	0.075***
	(0.001)	(0.003)	(0.002)	(0.002)
Single at baseline (γ_{kh})				
Men - Bottom Lifetime IQ	0	0	0	0
	(.)	(.)	(.)	(.)
Men - 2nd Lifetime IQ	-0.003*	-0.012***	0.007***	0.005**
	(0.002)	(0.005)	(0.002)	(0.002)
Men - 3rd Lifetime IQ	0.007***	-0.020***	-0.009***	-0.005**
	(0.002)	(0.005)	(0.002)	(0.002)
Men - 4th Lifetime IQ	0.014***	-0.024***	-0.026***	-0.018***
	(0.002)	(0.004)	(0.002)	(0.002)
Men - Top Lifetime IQ	0.026***	-0.016***	-0.027***	-0.027***
	(0.002)	(0.004)	(0.002)	(0.002)
Women - Bottom Lifetime IQ	0.003**	0.006*	-0.036***	-0.010***
	(0.001)	(0.003)	(0.002)	(0.002)
Women - 2nd Lifetime IQ	0.009***	0.002	-0.043***	-0.012***
	(0.001)	(0.004)	(0.002)	(0.002)
Women - 3rd Lifetime IQ	0.015***	-0.001	-0.051***	-0.022***
	(0.001)	(0.004)	(0.002)	(0.002)
Women - 4th Lifetime IQ	0.021***	0.002	-0.055***	-0.027***
	(0.001)	(0.004)	(0.002)	(0.002)
Women - Top Lifetime IQ	0.025***	0.017***	-0.056***	-0.033***
	(0.001)	(0.004)	(0.002)	(0.002)
Married at baseline (γ_{kh})				
Men - Bottom Lifetime IQ	0.026***	-0.012***	-0.044***	-0.021***
	(0.001)	(0.003)	(0.002)	(0.002)
Men - 2nd Lifetime IQ	0.038***	-0.011***	-0.048***	-0.028***
	(0.001)	(0.003)	(0.002)	(0.002)
Men - 3rd Lifetime IQ	0.044***	-0.011***	-0.047***	-0.029***
	(0.001)	(0.003)	(0.002)	(0.002)
Men - 4th Lifetime IQ	0.048***	-0.010***	-0.043***	-0.029***
	(0.001)	(0.003)	(0.002)	(0.002)
Men - Top Lifetime IQ	0.052***	-0.002	-0.039***	-0.028***
	(0.001)	(0.003)	(0.002)	(0.002)
Women - Bottom Lifetime IQ	0.014***	0.019***	-0.063***	-0.003*
	(0.001)	(0.004)	(0.002)	(0.002)
Women - 2nd Lifetime IQ	0.025***	0.001	-0.066***	-0.016***
	(0.001)	(0.004)	(0.002)	(0.002)
Women - 3rd Lifetime IQ	0.034***	-0.007**	-0.064***	-0.029***
	(0.001)	(0.004)	(0.002)	(0.002)
Women - 4th Lifetime IQ	0.041***	-0.000	-0.062***	-0.038***
	(0.001)	(0.004)	(0.002)	(0.002)
Women - Top Lifetime IQ	0.046***	0.012***	-0.057***	-0.043***
	(0.001)	(0.004)	(0.002)	(0.002)
Frailty: $\ln(\sigma^2)$	-16.369	0.261	-21.026	-2.104
Spells	4,028,551	4,028,551	1,795,027	1,795,027
Uncensored spells	1,425,236	206,997	770,070	622,346
Individuals	3,063,815	3,063,815	1,303,914	1,303,914
Sub-Log-likelihood (cf. (10))	-4,468,814.2	-1,055,077.0	-1,632,120.1	-1,530,497.4
Log-Likelihood	-5,523,891.2		-3,162,617.5	
Sub-Log-likelihood ($\sigma^2 = 0$)	-4,468,814.2	-1,057,575.3	-1,632,120.1	-1,538,180.9
Log-Likelihood ($\sigma^2 = 0$)	-5,526,389.5		-3,170,301.0	
LR test ($H_0 : \sigma^2 = 0$)	$p > 0.10$	$p < 0.01$	$p > 0.10$	$p < 0.01$

Significance levels: * 10-%; **5-%; ***1-%.
IQ = Income Quintile

F Additional Results

F.1 Life Expectancy and Long-term Care Use over Marital Status

Table 7: Remaining Life Expectancy and long-term care use at Age 65 by PI Quintiles

(a) Men	All	Bottom	Second	Third	Fourth	Top	ΔTop - Bottom
LE at age 65 (years)							
All	18.0 (17.9;18.1)	15.3 (15.0;15.5)	16.8 (16.6;17.0)	17.6 (17.5;17.8)	18.4 (18.2;18.6)	19.2 (19.1;19.4)	4.0 (3.7;4.2)
Initial Married	18.6 (18.4;18.7)	16.2 (15.9;16.5)	17.4 (17.2;17.6)	18.1 (17.9;18.3)	18.9 (18.7;19.1)	19.5 (19.3;19.8)	3.4 (3.0;3.7)
Initial Singles	16.1 (15.9;16.2)	14.1 (13.8;14.5)	14.7 (14.3;15.1)	15.5 (15.1;15.9)	16.4 (16.0;16.7)	17.9 (17.6;18.3)	3.8 (3.4;4.3)
LTC (years)⋆							
All	3.0 (3.0;3.1)	3.8 (3.7;4.0)	3.4 (3.3;3.6)	3.2 (3.1;3.2)	3.0 (2.9;3.0)	2.8 (2.7;2.8)	−1.1 (−1.2;−0.9)
Initial Married	2.9 (2.8;2.9)	3.0 (2.8;3.2)	3.1 (3.0;3.2)	3.0 (2.9;3.1)	2.9 (2.8;3.0)	2.7 (2.6;2.8)	−0.3 (−0.5;−0.1)
Initial Singles	3.7 (3.6;3.8)	4.8 (4.6;5.0)	4.6 (4.4;4.8)	3.8 (3.6;3.9)	3.2 (3.1;3.4)	2.9 (2.8;3.1)	−1.9 (−2.1;−1.6)
Ratio (%)							
All	12 (12;12)	20 (19;20)	15 (15;15)	12 (12;13)	11 (10;11)	9 (9;9)	−11 (−11;−10)
Initial Married	10 (9;10)	12 (11;12)	11 (11;11)	10 (10;10)	9 (9;10)	8 (8;9)	−4 (−4;−3)
Initial Singles	20 (20;21)	30 (29;31)	29 (28;30)	22 (21;22)	16 (16;17)	13 (12;13)	−17 (−18;−16)
Ever use LTC (%)							
All	77 (76;78)	79 (78;80)	79 (78;80)	78 (77;79)	77 (76;78)	75 (75;76)	−3 (−5;−2)
(b) Women							
LE at age 65 (years)							
All	21.9 (21.8;22.0)	20.1 (19.9;20.3)	21.8 (21.6;22.0)	22.0 (21.8;22.2)	22.2 (22.1;22.4)	22.3 (22.2;22.5)	2.3 (2.0;2.5)
Initial Married	22.5 (22.3;22.6)	22.1 (21.7;22.4)	22.5 (22.3;22.8)	22.4 (22.2;22.6)	22.6 (22.3;22.8)	22.5 (22.3;22.7)	0.4 (0.0;0.8)
Initial Singles	20.7 (20.6;20.8)	19.1 (18.9;19.3)	20.5 (20.2;20.8)	21.0 (20.8;21.3)	21.4 (21.1;21.7)	21.9 (21.6;22.1)	2.8 (2.4;3.1)
LTC (years)⋆							
All	5.1 (5.1;5.2)	6.0 (5.9;6.2)	5.9 (5.8;6.0)	5.3 (5.2;5.4)	4.8 (4.7;4.9)	4.4 (4.3;4.4)	−1.7 (−1.8;−1.5)
Initial Married	5.1 (5.1;5.2)	6.3 (6.1;6.6)	6.0 (5.9;6.2)	5.4 (5.3;5.5)	5.0 (4.8;5.1)	4.4 (4.3;4.6)	−1.9 (−2.1;−1.6)
Initial Singles	5.1 (5.0;5.1)	5.9 (5.8;6.0)	5.6 (5.5;5.8)	5.1 (5.0;5.3)	4.6 (4.4;4.7)	4.1 (3.9;4.2)	−1.8 (−2.0;−1.7)
Ratio (%)							
All	18 (18;18)	25 (25;26)	21 (21;22)	18 (18;19)	16 (16;16)	14 (13;14)	−12 (−12;−11)
Initial Married	16 (16;16)	21 (21;22)	20 (19;20)	17 (17;18)	15 (15;16)	13 (13;13)	−8 (−9;−8)
Initial Singles	21 (21;22)	27 (27;28)	24 (24;25)	21 (20;21)	18 (17;18)	15 (15;16)	−12 (−13;−12)
Ever use LTC (%)							
All	89 (88;89)	91 (91;92)	91 (91;92)	89 (89;90)	88 (88;89)	86 (85;87)	−5 (−6;−4)

Notes: These are population-averaged measures for the life cycle simulation of 100,000 households. We present the median estimates across 5,000 bootstrapped samples and the 2.5^{th} and 97.5^{th} percentiles between brackets.

F.2 Premium Returns

The following tables provide the data to Figures 2 and 4 presented in the main text.

Table 8: Premium returns for different groups (in %)

Income quintile:	Bottom	Second	Third	Fourth	Top
Household level ($\rho^* = 1.35$)					
Pension Annuity	−8.9	−2.6	−0.6	1.5	3.6
	(−9.6;−8.2)	(−3.2;−1.9)	(−1.1;−0.1)	(1.0;2.0)	(3.2;4.0)
LTC insurance	29.9	17.9	4.1	−6.0	−17.0
	(27.6;31.9)	(16.0;19.7)	(2.7;5.6)	(−7.4;−4.7)	(−18.2;−15.8)
Life care annuity	−1.4	1.4	0.3	0.1	−0.3
	(−2.2;−0.8)	(0.6;2.0)	(−0.4;0.9)	(−0.5;0.7)	(−0.7;0.0)
Single Men ($\rho^* = 2.11$)					
Pension Annuity	−12.0	−8.3	−3.3	1.9	11.8
	(−13.9;−10.1)	(−10.7;−5.9)	(−5.5;−1.3)	(0.1;3.7)	(10.1;13.4)
LTC insurance	29.8	28.9	2.3	−13.8	−21.6
	(24.9;34.8)	(23.0;34.7)	(−2.0;6.7)	(−17.4;−10.0)	(−25.0;−18.3)
Life care annuity	0.0	2.4	−1.7	−2.6	2.2
	(−2.0;1.9)	(−0.3;4.9)	(−4.3;0.8)	(−4.6;−0.4)	(0.8;3.8)
Single Women ($\rho^* = 1.47$)					
Pension Annuity	−7.7	−0.7	1.7	3.4	5.6
	(−8.6;−6.8)	(−2.0;0.5)	(0.5;3.0)	(2.2;4.5)	(4.5;6.9)
LTC insurance	16.0	12.5	1.1	−10.7	−20.8
	(13.8;18.3)	(9.4;15.6)	(−1.7;4.0)	(−13.2;−8.1)	(−23.2;−18.2)
Life care annuity	−1.7	2.6	1.6	−0.1	−1.0
	(−2.6;−0.9)	(1.1;4.1)	(0.1;3.1)	(−1.5;1.2)	(−1.9;0.1)
Married Men ($\rho^* = 11.16$)					
Pension Annuity	−12.9	−6.2	−2.4	1.7	5.3
	(−14.5;−11.4)	(−7.3;−5.2)	(−3.2;−1.5)	(0.9;2.4)	(4.6;6.0)
LTC insurance	4.2	8.5	5.2	0.1	−8.0
	(−1.9;10.1)	(4.1;12.9)	(2.0;8.6)	(−2.7;3.0)	(−10.4;−5.3)
Life care annuity	−3.1	2.1	2.0	0.8	−2.3
	(−6.5;−0.9)	(−0.7;4.7)	(−0.2;4.1)	(−1.1;2.7)	(−3.5;−0.8)
Married women ($\rho^* = 0.08$)					
Pension Annuity	−1.7	0.3	−0.3	0.4	0.0
	(−3.1;−0.4)	(−0.7;1.3)	(−1.1;0.5)	(−0.3;1.1)	(−0.6;0.7)
LTC insurance	26.8	21.1	7.0	−3.4	−15.9
	(22.1;31.6)	(17.8;24.3)	(4.6;9.5)	(−5.5;−1.2)	(−17.8;−14.0)
Life care annuity	−1.3	0.6	−0.2	0.4	−0.2
	(−2.6;0.0)	(−0.2;1.4)	(−0.9;0.6)	(−0.4;1.1)	(−0.6;0.3)

Notes: The premium returns are population-averaged measures for the life cycle simulation of 100,000 individuals. Medians across 5,000 bootstrapped samples and the $2.5^{\text{th}}$and 97.5^{th} percentile (in brackets) are shown.